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Jarasandha Numbers

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Preface

Number is the essence of mathematical calculations. Numbers have varieties of patterns and have varieties of range and richness. Many numbers exhibit fascinating properties, they form sequences, they form patterns and so on. A source of interest to both amateur and professional number theorist is the study of patterns in integers and they can be studied both geometrically and algebraically. A vital problem-solving skill is the recognition of patterns in integers. Recognizing number patterns is a vital problem-solving skill. As noted by the Annenburg Foundation, “If you see a pattern when you look systematically at specific examples, you can use that pattern to generalize what you see into a broader solution to a problem”. Understanding number patterns are necessary so that students of all ages can appropriately identify and understand various types of patterns and functional relationships. Furthermore, number pattern awareness allows one to use patterns and models to analyze the change in both real and abstract contexts. In this context, for simplicity and brevity, one may refer [Carmichael.,1959, Chris Brink.,2025, Conway, Guy.,2006, Dickson.,1952, Gopalan et.al.,2025, Maheswari, Devibala.,2023, Malini Devi, Devibala.,2021. Mordell.,1969, Pandichelvi, Vanaja.,2024, Shailesh Shirali.,2001, Thiruniraiselvi, Gopalan.,2021, Thiruniraiselvi et.al.,2025, Vijayasankar et.al.,2019].

In our Indian epic Mahabharata, we come across a demoniac figure named Jarasandha. He had a boon that if he was split into two parts and thrown apart, the parts would rejoin and return to life. In fact, he was given life by the two halves of his body. In the field of Mathematics, we have numbers exhibiting the same property as Jarasandha.

This book focuses on categorizing special polygonal numbers, pyramidal numbers and geometrical representations in relation to Jarasandha numbers. This book contains a reasonable collection of special polygonal and pyramidal numbers. The procedure in obtaining various relations are illustrated in an elegant manner.

Dr. S. Devibala
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Notations

Polygonal number of rank n with size $m = t_{m,n} = n \left[1 + \frac{(n-1)(m-2)}{2} \right]$

Centered Polygonal number of rank n with size $m = Ct_{m,n} = m t_{m,n} + 1$

Pyramidal number of rank n with size $m = P_n^m = \frac{n(n+1)}{6} [(m-2)n + (5-m)]$

Chapter 1: Basic Definition of Jarasandha Number with Illustrations

Definition 1.1: Jarasandha Number

Let N be a positive number which may be split as two numbers X and C .

If these numbers are added and squared, we get the same number XC again.

$$\text{That is } (X+C)^2 = XC = N \tag{1.1}$$

Here, N is called Jarasandha number.

Remark 1.2

If C is a single digit number then write

$$XC = 10 \cdot X + C \tag{1.2}$$

In general, if C is a n -digit number, then write

$$XC = 10^n \cdot X + C \tag{1.3}$$

To obtain numerical examples of Jarasandha numbers, we proceed as shown below:

Example 1

Let $C = 1$

In view of (1.2), we have

$$\begin{aligned} (X+1)^2 &= 10X+1 \\ \Rightarrow X^2 - 8X &= 0 \\ \Rightarrow X &= 0, 8 \end{aligned}$$

Thus, the Jarasandha numbers are 01, 81

The Jarasandha number 01 is a trivial example

Example 2

Let $C = 01$

In view of (1.3), we have

$$\begin{aligned}(X + 01)^2 &= 10^2 X + 01 \\ \Rightarrow X^2 - 98X &= 0 \\ \Rightarrow X &= 0,98\end{aligned}$$

Thus, the non-trivial Jarasandha number is 9801

Example 3

Let $C = 001$

In view of (1.3), we have

$$\begin{aligned}(X + 001)^2 &= 10^3 X + 001 \\ \Rightarrow X^2 - 998X &= 0 \\ \Rightarrow X &= 0,998\end{aligned}$$

Thus, the non-trivial Jarasandha number is 998001

Example 4

Let $C = 25$

In view of (3), we get

$$\begin{aligned}(X + 25)^2 &= 10^2 X + 25 \\ \Rightarrow X^2 - 50X + 600 &= 0 \\ \Rightarrow X &= 20,30\end{aligned}$$

Thus, the Jarasandha numbers are 2025, 3025

Example 5

Let $C = 209$

In view of (1.3), we get

$$\begin{aligned}
(X + 209)^2 &= 10^3 X + 209 \\
\Rightarrow X^2 + (2 * 209 - 10^3)X + 209^2 - 209 &= 0 \\
\Rightarrow X^2 - 582X + 43472 &= 0 \\
\Rightarrow X &= 88,494
\end{aligned}$$

Thus, the Jarasandha numbers are 88209 ,494209

Example 6

$$\text{Let } C = 1729$$

In view of (1.3), we get

$$\begin{aligned}
(X + 1729)^2 &= 10^4 X + 1729 \\
\Rightarrow X^2 + (2 * 1729 - 10^4)X + 1729^2 - 1729 &= 0 \\
\Rightarrow X^2 - 6542X + 2987712 &= 0 \\
\Rightarrow X &= 6048,494
\end{aligned}$$

Thus, the Jarasandha numbers are 60481729, 4941729

Example 7

$$\text{Let } C = 1984$$

In view of (1.3), we have

$$\begin{aligned}
(X + 1984)^2 &= 10^4 X + 1984 \\
\Rightarrow X^2 + (2 * 1984 - 10^4)X + 1984^2 - 1984 &= 0 \\
\Rightarrow X^2 - 6032X + 3934272 &= 0 \\
\Rightarrow X &= 744,5288
\end{aligned}$$

Thus, the Jarasandha numbers are 7441984 ,52881984

Example 8

$$\text{Let } C = 2500$$

In view of (1.3), we have

$$\begin{aligned}
(X + 2500)^2 &= 10^4 X + 2500 \\
\Rightarrow X^2 + (2 * 2500 - 10^4)X + 2500^2 - 2500 &= 0 \\
\Rightarrow X^2 - 5000X + 6247500 &= 0 \\
\Rightarrow X &= 2550,2450
\end{aligned}$$

Thus, the Jarasandha numbers are 25502500 , 24502500

Example 9

Let $C = 17284$

In view of (1.3), we have

$$\begin{aligned}(X+17284)^2 &= 10^5 X + 17284 \\ \Rightarrow X^2 + (2*17284 - 10^5)X + 17284^2 - 17284 &= 0 \\ \Rightarrow X^2 - 65432X + 17284*17283 &= 0 \\ \Rightarrow X &= 60494,4938\end{aligned}$$

Thus, the Jarasandha numbers are 6049417284, 493817284

Example 10

Let $C = 04641$

In view of (1.3), we have

$$\begin{aligned}(X+04641)^2 &= 10^5 X + 04641 \\ \Rightarrow X^2 + (2*04641 - 10^5)X + 04641^2 - 04641 &= 0 \\ \Rightarrow X^2 - 90718X + 21534240 &= 0 \\ \Rightarrow X &= 90480,238\end{aligned}$$

Thus, the Jarasandha numbers are 9048004641 , 23804641

Example 11

Let $C = 14336$

In view of (1.3), we have

$$\begin{aligned}(X+14336)^2 &= 10^5 X + 14336 \\ \Rightarrow X^2 + (2*14336 - 10^5)X + 14336^2 - 14336 &= 0 \\ \Rightarrow X^2 - 71328X + 205506560 &= 0 \\ \Rightarrow X &= 68320,3008\end{aligned}$$

Thus, the Jarasandha numbers are 6832014336, 300814336.

Chapter 2: Linear Combination of Two Polygonal Numbers of Same Size and of Different Ranks in Connection with Special Jarasandha Numbers

Connection 2.1

The assumption

$$t_{3,n} - t_{3,m} = 81 \tag{2.1}$$

is equivalent to

$$\begin{aligned} n(n+1) - m(m+1) &= 162 \\ \Rightarrow (n-m) + (n^2 - m^2) &= 162 \\ \Rightarrow (n-m)(n+m+1) &= 162 \end{aligned} \tag{2.2}$$

The values of n, m satisfying (2.2) and the corresponding values of $t_{3,n}, t_{3,m}$ satisfying (2.1) are given in Table 2.1 below:

Table 2.1: Values of $n, m, t_{3,n}, t_{3,m}$

n	m	$t_{3,n}$	$t_{3,m}$	Jarasandha number
81	80	3321	3240	81
41	39	861	780	81
28	25	406	325	81
16	10	136	55	81
13	4	91	10	81

Connection 2.2

The assumption

$$t_{3,n} - t_{3,m} = 2025 \quad (2.3)$$

is equivalent to

$$\begin{aligned} n(n+1) - m(m+1) &= 4050 \\ \Rightarrow (n-m) + (n^2 - m^2) &= 4050 \\ \Rightarrow (n-m)(n+m+1) &= 4050 \end{aligned} \quad (2.4)$$

The values of n, m satisfying (2.4) and the corresponding values of $t_{3,n}, t_{3,m}$ satisfying (2.3) are given in Table 2.2 below:

Table 2.2: values of $n, m, t_{3,n}, t_{3,m}$

n	m	$t_{3,n}$	$t_{3,m}$	Jarasandha number
2025	2024	2051325	2049300	2025
1013	1011	513591	511566	2025
676	673	228826	226801	2025
407	402	83028	81003	2025
340	334	57970	55945	2025
229	220	26335	24310	2025
207	197	21528	19503	2025
142	127	10153	8128	2025
121	103	7381	5356	2025
93	68	4371	2346	2025
88	61	3916	1891	2025
82	52	3403	1378	2025
67	22	2278	253	2025
65	15	2145	120	2025
64	10	2080	55	2025

Connection 2.3

The assumption

$$t_{3,n} - t_{3,m} = 3025 \quad (2.5)$$

is equivalent to

$$\begin{aligned}
n(n+1) - m(m+1) &= 6050 \\
\Rightarrow (n-m) + (n^2 - m^2) &= 6050 \\
\Rightarrow (n-m)(n+m+1) &= 6050
\end{aligned}
\tag{2.6}$$

The values of n, m satisfying (2.6) and the corresponding values of $t_{3,n}, t_{3,m}$ satisfying (2.5) are given in Table 2.3 below:

Table 2.3: values of $n, m, t_{3,n}, t_{3,m}$

n	m	$t_{3,n}$	$t_{3,m}$	Jarasandha number
3025	3024	4576825	4573800	3025
1513	1511	1145341	1142316	3025
607	602	184528	181503	3025
307	297	47278	44253	3025
280	269	39340	36315	3025
148	126	11026	8001	3025
133	108	8911	5886	3025
85	35	3655	630	3025
82	27	3403	378	3025

Connection 2.4

The assumption

$$t_{4,n} - t_{4,m} = 81 \tag{2.7}$$

is equivalent to

$$(n^2 - m^2) = 81 \tag{2.8}$$

The values of n, m satisfying (2.8) and the corresponding values of $t_{4,n}, t_{4,m}$ satisfying (2.7) are given in Table 2.4 below:

Table 2.4: values of $n, m, t_{4,n}, t_{4,m}$

n	m	$t_{4,n}$	$t_{4,m}$	Jarasandha number
41	40	1681	1600	81
15	12	225	144	81

Connection 2.5

The assumption

$$t_{4,n} - t_{4,m} = 2025 \quad (2.9)$$

is equivalent to

$$(n^2 - m^2) = 2025 \quad (2.10)$$

The values of n, m satisfying (2.10) and the corresponding values of $t_{4,n}, t_{4,m}$ satisfying (2.9) are given in Table 2.5 below:

Table 2.5: values of $n, m, t_{4,n}, t_{4,m}$

n	m	$t_{4,n}$	$t_{4,m}$	Jarasandha number
1013	1012	1026169	1024144	2025
339	336	114921	112896	2025
205	200	42025	40000	2025
117	108	13689	11664	2025
75	60	5625	3600	2025
53	28	2809	784	2025
51	24	2601	576	2025

Connection 2.6

The assumption

$$t_{4,n} - t_{4,m} = 3025 \quad (2.11)$$

is equivalent to

$$(n^2 - m^2) = 3025 \quad (2.12)$$

The values of n, m satisfying (2.12) and the corresponding values of $t_{4,n}, t_{4,m}$ satisfying (2.11) are given in Table 2.6 below:

Table 2.6: values of $n, m, t_{4,n}, t_{4,m}$

n	m	$t_{4,n}$	$t_{4,m}$	$t_{4,n} - t_{4,m}$
1513	1512	2289169	2286144	3025
305	300	93025	90000	3025
143	132	20449	17424	3025
73	48	5329	2304	3025

Connection 2.7

The assumption

$$t_{10,n} - t_{10,m} = 2025 \quad (2.13)$$

is equivalent to

$$(n - m)(4n + 4m - 3) = 2025 \quad (2.14)$$

The values of n, m satisfying (2.14) and the corresponding values of $t_{10,n}, t_{10,m}$ satisfying (2.13) are given in Table 2.7 below:

Table 2.7: values of $n, m, t_{10,n}, t_{10,m}$

n	m	$t_{10,n}$	$t_{10,m}$	$t_{10,n} - t_{10,m}$
254	253	257302	255277	2025
33	24	4257	2232	2025

Chapter 3: Linear Combination of Two Centered Polygonal Numbers of Same Size and of Different Ranks in Connection with Special Jarasandha Numbers

Connection 3.1

The assumption

$$Ct_{3,n} - Ct_{3,m} = 81 \tag{3.1}$$

is equivalent to

$$\begin{aligned} t_{3,n} - t_{3,m} &= 27 \\ \Rightarrow n(n+1) - m(m+1) &= 54 \\ \Rightarrow (n-m)(n+m+1) &= 54 \end{aligned} \tag{3.2}$$

The values of n, m satisfying (3.2) and the corresponding values of $Ct_{3,n}, Ct_{3,m}$ satisfying (3.1) are given in Table 3.1 below:

Table 3.1: values of $n, m, Ct_{3,n}, Ct_{3,m}$

n	m	$Ct_{3,n}$	$Ct_{3,m}$	$Ct_{3,n} - Ct_{3,m}$
27	26	1135	1054	81
14	12	316	235	81
10	7	166	85	81
7	1	85	4	81

Connection 3.2

The assumption

$$Ct_{4,n} - Ct_{4,m} = \text{Four times the two digits Jarasandha number } 81 \quad (3.3)$$

is equivalent to

$$\begin{aligned} \Rightarrow 2n(n+1) - 2m(m+1) &= 4 * 81 \\ \Rightarrow (n-m)(n+m+1) &= 162 \end{aligned} \quad (3.4)$$

The values of n, m satisfying (3.4) and the corresponding values of $Ct_{4,n}, Ct_{4,m}$ satisfying (3.3) are given in Table 3.2 below:

Table 3.2: values of $n, m, Ct_{4,n}, Ct_{4,m}$

n	m	$Ct_{4,n}$	$Ct_{4,m}$	$\frac{Ct_{4,n} - Ct_{4,m}}{4}$
81	80	13285	12961	81
41	39	3445	3121	81
28	25	1625	1301	81
16	10	545	221	81
13	4	365	41	81

Connection 3.3

The assumption

$$Ct_{5,n} - Ct_{5,m} = 2025 \quad (3.5)$$

is equivalent to

$$\begin{aligned} \Rightarrow 5n(n+1) - 5m(m+1) &= 4050 \\ \Rightarrow (n-m)(n+m+1) &= 810 \end{aligned} \quad (3.6)$$

The values of n, m satisfying (3.6) and the corresponding values of $Ct_{5,n}, Ct_{5,m}$ satisfying (3.5) are given in Table 3.3 below:

Table 3.3: values of $n, m, Ct_{5,n}, Ct_{5,m}$

n	m	$Ct_{5,n}$	$Ct_{5,m}$	$Ct_{5,n} - Ct_{5,m}$
405	404	411076	409051	2025
203	201	103531	101506	2025
136	133	46581	44556	2025
83	78	17431	15406	2025
70	64	12426	10401	2025
49	40	6126	4101	2025
45	35	5176	3151	2025
34	19	2976	951	2025
28	1	2031	6	2025

Connection 3.4

The assumption

$$Ct_{11,n} - Ct_{11,m} = 3025 \quad (3.7)$$

is equivalent to

$$\begin{aligned} \Rightarrow 11n(n+1) - 11m(m+1) &= 6050 \\ \Rightarrow (n-m)(n+m+1) &= 550 \end{aligned} \quad (3.8)$$

The values of n, m satisfying (3.8) and the corresponding values of $Ct_{11,n}, Ct_{11,m}$ satisfying (3.7) are given in Table 3.4 below:

Table 3.4: values of $n, m, Ct_{11,n}, Ct_{11,m}$

n	m	$Ct_{11,n}$	$Ct_{11,m}$	$Ct_{11,n} - Ct_{11,m}$
275	274	417451	414426	3025
138	136	105502	102477	3025
57	52	18184	15159	3025
32	22	5809	2784	3025
30	19	5116	2091	3025
23	1	3037	12	3025

Connection 3.5

The assumption

$$Ct_{12,n} - Ct_{12,m} = 4 * 81 \tag{3.9}$$

is equivalent to

$$\begin{aligned} \Rightarrow 6n(n+1) - 6m(m+1) &= 324 \\ \Rightarrow (n-m)(n+m+1) &= 54 \end{aligned} \tag{3.10}$$

The values of n, m satisfying (3.10) and the corresponding values of $Ct_{11,n}, Ct_{11,m}$ satisfying (3.9) are given in Table 3.5 below:

Table 3.5: values of $n, m, Ct_{12,n}, Ct_{12,m}$

n	m	$Ct_{12,n}$	$Ct_{12,m}$	$\frac{Ct_{12,n} - Ct_{12,m}}{4}$
27	26	4537	4213	81
14	12	1261	937	81
10	7	661	337	81
7	1	337	13	81

Chapter 4: Linear Combination of Two Pyramidal Numbers of Same Size and of Different Ranks in Connection with Special Jarasandha Numbers

Relation 4.1

The assumption

$$\left(\frac{3P_n^3}{t_{3,n}}\right)^2 - \left(\frac{3P_m^3}{t_{3,m}}\right)^2 = 81 \tag{4.1}$$

is equivalent to

$$(n - m)(n + m + 4) = 81 \tag{4.2}$$

The values of n, m satisfying (4.2) and the corresponding values of $t_{3,n}, t_{3,m}, P_n^3, P_m^3$ satisfying (4.1) are given in Table 1 below:

Table 4.1: values of $n, m, t_{3,n}, t_{3,m}, P_n^3, P_m^3$

n	m	$t_{3,n}$	$t_{3,m}$	P_n^3	P_m^3	$\left(\frac{3P_n^3}{t_{3,n}}\right)^2 - \left(\frac{3P_m^3}{t_{3,m}}\right)^2$
39	38	780	741	10660	9880	$41^2 - 40^2 = 81$
13	10	91	55	455	220	$15^2 - 12^2 = 81$

Relation 4.2

The assumption

$$\left(\frac{3P_n^3}{t_{3,n}}\right)^2 - \left(\frac{3P_m^3}{t_{3,m}}\right)^2 = \text{Three times the two digits Jarasandha number } 81 \tag{4.3}$$

is equivalent to

$$(n - m)(n + m + 4) = 243 \quad (4.4)$$

The values of n, m satisfying (4.4) and the corresponding values of $t_{3,n}, t_{3,m}, P_n^3, P_m^3$ satisfying (4.3) are given in Table 4.2 below:

Table 4. 2: values of $n, m, t_{3,n}, t_{3,m}, P_n^3, P_m^3$

n	m	$t_{3,n}$	$t_{3,m}$	P_n^3	P_m^3	$\left(\frac{3P_n^3}{t_{3,n}}\right)^2 - \left(\frac{3P_m^3}{t_{3,m}}\right)^2$
120	119	7260	7140	295240	287980	$122^2 - 121^2 = 243$
40	37	820	703	11480	9139	$42^2 - 39^2 = 243$
16	7	136	28	816	84	$18^2 - 9^2 = 243$

Relation 4.3

The assumption

$$\left(\frac{3P_n^3}{t_{3,n}}\right)^2 - \left(\frac{3P_m^3}{t_{3,m}}\right)^2 = \text{Four digits Jarasandha number 2025} \quad (4.5)$$

is equivalent to

$$(n - m)(n + m + 4) = 2025 \quad (4.6)$$

The values of n, m satisfying (4.6) and the corresponding values of $t_{3,n}, t_{3,m}, P_n^3, P_m^3$ satisfying (4.5) are given in Table 4.3 below:

Table 4.3: values of $n, m, t_{3,n}, t_{3,m}, P_n^3, P_m^3$

n	m	$t_{3,n}$	$t_{3,m}$	P_n^3	P_m^3	$\left(\frac{3P_n^3}{t_{3,n}}\right)^2 - \left(\frac{3P_m^3}{t_{3,m}}\right)^2$
1011	1010	511566	510555	172738786	172227220	$1013^2 - 1012^2 = 2025$
337	334	56953	55945	6435689	6265840	$339^2 - 336^2 = 2025$
203	198	20706	19701	1414910	1313400	$205^2 - 200^2 = 2025$
115	106	6670	5671	260130	204156	$117^2 - 108^2 = 2025$
73	58	2701	1711	67525	34220	$75^2 - 60^2 = 2025$

Relation 4.4

The assumption

$$\left(\frac{3P_n^3}{t_{3,n}}\right)^2 - \left(\frac{3P_m^3}{t_{3,m}}\right)^2 = \text{Four digits Jarasandha number } 9801 \quad (4.7)$$

is equivalent to

$$(n - m)(n + m + 4) = 2025 \quad (4.8)$$

The values of n, m satisfying (4.8) are given in Table 4.4 below:

Table 4.4: values of n, m

n	m
4899	4898
1633	1630
547	538
449	438
163	130

The readers can check that (4.7) is satisfied by the values of n and m given in the above Table 4.4.

Relation 4.5

The assumption

$$\left(\frac{2P_n^5}{t_{4,n}}\right)^2 - \left(\frac{2P_m^5}{t_{3,m}}\right)^2 = 81 \quad (4.9)$$

is equivalent to

$$(n - m)(n + m + 2) = 81 \quad (4.10)$$

The values of n, m satisfying (4.10) and the corresponding values of $t_{4,n}, t_{4,m}, P_n^5, P_m^5$ satisfying (4.9) are given in Table 5 below:

Table4. 5: values of $n, m, t_{4,n}, t_{4,m}, P_n^5, P_m^5$

n	m	$t_{4,n}$	$t_{4,m}$	P_n^5	P_m^5	$\left(\frac{2P_n^5}{t_{4,n}}\right)^2 - \left(\frac{2P_m^5}{t_{4,m}}\right)^2$
40	39	1600	1521	32800	30420	$41^2 - 40^2 = 81$
14	11	196	121	1470	726	$15^2 - 12^2 = 81$

Relation 4.6

The assumption

$$\left(\frac{2P_n^5}{t_{4,n}}\right)^2 - \left(\frac{2P_m^5}{t_{3,m}}\right)^2 = 2025 \quad (4.11)$$

is equivalent to

$$(n - m)(n + m + 2) = 2025 \quad (4.12)$$

The values of n, m satisfying (4.12) and the corresponding values of $t_{4,n}, t_{4,m}, P_n^5, P_m^5$ satisfying (4.11) are given in Table 4.6 below:

Table 4.6: values of $n, m, t_{4,n}, t_{4,m}, P_n^5, P_m^5$

n	m	$t_{4,n}$	$t_{4,m}$	P_n^5	P_m^5	$\left(\frac{2P_n^5}{t_{4,n}}\right)^2 - \left(\frac{2P_m^5}{t_{4,m}}\right)^2$
1012	1011	1024144	1022121	518728936	517193226	$1013^2 - 1012^2 = 2025$
338	335	114244	112225	19364358	18853800	$339^2 - 336^2 = 2025$
204	199	41616	39601	4265640	3960100	$205^2 - 200^2 = 2025$
116	107	13456	11449	787176	618246	$117^2 - 108^2 = 2025$
74	59	5476	3481	205350	104430	$75^2 - 60^2 = 2025$
52	27	2704	729	71656	10206	$53^2 - 28^2 = 2025$
50	23	2500	529	63750	6348	$51^2 - 24^2 = 2025$

Relation 4.7

The assumption

$$\left(\frac{2P_n^5}{t_{4,n}}\right)^2 - \left(\frac{2P_m^5}{t_{3,m}}\right)^2 = 3025 \quad (4.13)$$

is equivalent to

$$(n - m) (n + m + 2) = 3025 \quad (4.14)$$

The values of n, m satisfying (4.14) and the corresponding values of $t_{4,n}, t_{4,m}, P_n^5, P_m^5$ satisfying (4.13) are given in Table 4.7 below:

Table 4.7: values of $n, m, t_{4,n}, t_{4,m}, P_n^5, P_m^5$

n	m	$t_{4,n}$	$t_{4,m}$	P_n^5	P_m^5	$\left(\frac{2P_n^5}{t_{4,n}}\right)^2 - \left(\frac{2P_m^5}{t_{4,m}}\right)^2$
1512	1511	2286144	2283121	1729467936	1726039476	$1513^2 - 1512^2 = 3025$
304	299	92416	89401	14093440	13410150	$305^2 - 300^2 = 3025$
142	131	20164	17161	1441726	1132626	$143^2 - 132^2 = 3025$
72	47	5184	2209	189216	53016	$73^2 - 48^2 = 3025$

Relation 4.8

The assumption

$$\left(\frac{2P_n^5}{t_{4,n}}\right)^2 - \left(\frac{2P_m^5}{t_{3,m}}\right)^2 = 9801 \quad (4.15)$$

is equivalent to

$$(n - m) (n + m + 2) = 9801 \quad (4.16)$$

The values of n, m satisfying (4.16) and the corresponding values of $t_{4,n}, t_{4,m}, P_n^5, P_m^5$ satisfying (4.15) are given in Table 4.8 below:

Table 4.8: values of $n, m, t_{4,n}, t_{4,m}, P_n^5, P_m^5$

n	m
4900	4899
1634	1631
548	539
450	439
194	167
164	131

100	19
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The readers can check that (4.15) is satisfied by the values of n and m given in the above Table 4.8.

Chapter 5: Characterizations of Pythagorean Triangles in Relation to Special Jarasandha Numbers

Let p, q be two non-zero distinct positive integers such that $p > q > 0$.

Considering p, q to be the generators of a Pythagorean triangle, its legs denoted by x, y and hypotenuse H are taken as

$$x = 2pq, y = p^2 - q^2, H = p^2 + q^2 \tag{5.1}$$

Denoting the area and the perimeter of the above Pythagorean triangle as A, P respectively, one has

$$A = pq(p^2 - q^2), P = 2p(p + q) \tag{5.2}$$

At the outset, it is worth to remind that the area of Pythagorean triangle (3,4,5) is six and that, the area of Pythagorean triangle ($3*a, 4*a, 5*a$) is $6*a^2$.

Now, It is well-known by definition that, Jarasandha numbers are plenty and each Jarasandha number is a perfect square. Thus, it is observed that, six times the Jarasandha number represents the area of a Pythagorean triangle.

Illustration 1

Consider the Jarasandha number 81.

$6*81$ is the area of Pythagorean triangle, whose legs and hypotenuse are respectively $3*9, 4*9, 5*9$

Illustration 2

Consider the Jarasandha number 88209.

$6*88209$ is the area of Pythagorean triangle, whose legs and hypotenuse are respectively $3*297, 4*297, 5*297$

In what follows, we illustrate the process of other characterizations of Pythagorean triangles in relation to special Jarasandha numbers.

Characterization 5.0

(i) Consider the Jarasandha number denoted by

$$J_1=2025=45^2= 9^2*25$$

Now,

$$J_1^2=9^4*25^2$$

We know that

$$25^2=7^2+24^2=15^2+20^2 \tag{*}$$

In view of the above relation, we see that

$$\begin{aligned} J_1^2 &= 9^4 * (7^2 + 24^2) = 9^4 * (15^2 + 20^2) \\ &= (81 * 7)^2 + (81 * 24)^2 = (81 * 15)^2 + (81 * 20)^2 \\ &= (567^2 + 1944^2) = (1215^2 + 1620^2) \end{aligned}$$

Thus, it is observed that, J_1 represents hypotenuse of two Pythagorean triangles given by $(567, 1944, 2025)$ & $(1215, 1620, 2025)$.

Further, observe that, J_1^2 is a second order Ramanujan number as it is equivalent to sum of two squares in two different ways.

(ii) Consider the Jarasandha number denoted by

$$J_2=3025=55^2= 11^2*25$$

Now,

$$J_2^2=11^4*25^2$$

In view of (*), we see that

$$\begin{aligned} J_2^2 &= 11^4 * (7^2 + 24^2) = 11^4 * (15^2 + 20^2) \\ &= (121 * 7)^2 + (121 * 24)^2 = (121 * 15)^2 + (121 * 20)^2 \\ &= (847^2 + 2904^2) = (1815^2 + 2420^2) \end{aligned}$$

Thus, it is observed that, J_2 represents hypotenuse of two Pythagorean triangles given by (847, 2904, 3025) & (1815, 2420, 3025).

Further, observe that, J_2^2 is a second order Ramanujan number as it is equivalent to sum of two squares in two different ways.

(iii) Consider the Jarasandha number denoted by

$$J_3=24502500=4950^2= 990^2*25$$

Now,

$$J_3^2=990^4*25^2$$

In view of (*), we see that

$$\begin{aligned} J_3^2 &= 990^4*(7^2+24^2) = 990^4*(15^2+20^2) \\ &= (990^2*7)^2+(990^2*24)^2 = (990^2*15)^2+(990^2*20)^2 \end{aligned}$$

Thus, it is observed that, J_1 represents hypotenuse of two Pythagorean triangles given by $(990^2*7, 990^2*24, J_3)$ & $(990^2*15, 990^2*20, J_3)$.

Further, observe that, J_3^2 is a second order Ramanujan number as it is equivalent to sum of two squares in two different ways.

(iv) Consider the Jarasandha number denoted by

$$J_4=25502500=5050^2= 1010^2*25$$

Now,

$$J_4^2=1010^4*25^2$$

In view of (*), we see that

$$\begin{aligned} J_4^2 &= 1010^4*(7^2+24^2) = 1010^4*(15^2+20^2) \\ &= (1010^2*7)^2+(1010^2*24)^2=(1010^2*15)^2+(1010^2*20)^2 \end{aligned}$$

Thus, it is observed that, J_1 represents hypotenuse of two Pythagorean triangles given by $(1010^2*7, 1010^2*24, J_4)$ & $(1010^2*15, 1010^2*20, J_4)$.

Further, observe that, J_4^2 is a second order Ramanujan number as it is equivalent to sum of two squares in two different ways.

Characterization 5.1

Assume that two times the ratio of area by perimeter of a Pythagorean triangle is represented by Jarasandha number 81. From (5.2), the corresponding equation to be solved is

$$q(p - q) = 81 \tag{5.3}$$

After performing a few calculations, the values of p, q satisfying (5.3) and the sides of corresponding Pythagorean triangle are presented in Table - 5.1 below:

Table- 5.1-Pythagorean triangle with $2\frac{A}{P} = 81$

p	q	x	y	H	$2\frac{A}{P}$
82	1	164	6723	6725	81
	81	13284	163	13285	81
30	3	180	891	909	81
	27	1620	171	1629	81
18	9	324	243	405	81

Characterization 5.2

Assume

$$2y - x - H = 2025 \tag{5.4}$$

From (5.1) , the corresponding equation to be solved is

$$\begin{aligned} 2(p^2 - q^2) - 2pq - (p^2 + q^2) &= 2025 \\ \Rightarrow p^2 - 2pq - 3q^2 &= 2025 \\ \Rightarrow (p + q)(p - 3q) &= 2025 \end{aligned} \tag{5.5}$$

After performing a few calculations, the values of p,q satisfying (5.5) and the sides of corresponding Pythagorean triangle are presented in Table-5.2 below:

Table -5.2-Pythagorean triangle with $2y - x - H = 2025$

p	q	x	y	H	$2y - x - H$
1519	506	1537228	2051325	2563397	2025
507	168	170352	228825	285273	2025
305	100	61000	83025	103025	2025
171	54	18468	26325	32157	2025
105	30	6300	10125	11925	2025
67	14	1876	4293	4685	2025
63	12	1512	3825	4113	2025

Characterization 5.3

Assume

$$H - 4\frac{A}{P} - 6q^2 = 81 \quad (5.6)$$

From (5.1) & (5.2), the corresponding equation to be solved is

$$\begin{aligned} p^2 + q^2 - 2q(p - q) - 6q^2 &= 81 \\ \Rightarrow p^2 - 2pq - 3q^2 &= 81 \\ \Rightarrow (p + q)(p - 3q) &= 81 \end{aligned} \quad (5.7)$$

After performing a few calculations ,the values of p ,q satisfying (5.7) and the sides of the corresponding Pythagorean triangle are presented in Table- 5.3 below:

Table-5.3- Pythagorean triangle with $H - 4\frac{A}{P} - 6q^2 = 81$

p	q	x	y	H	$H - 4\frac{A}{P} - 6q^2$
61	20	2440	3321	4121	81
21	6	252	405	477	81

It is to be noticed from (5.1) that

$$3(y - H) = -6q^2 .$$

Thus, we have the characterization of the Pythagorean triangle as

$$H - 4\frac{A}{P} + 3(y - H) = 81$$

Characterization 5.4

Assume

$$y - 3x + \frac{(H - y)}{2} = 2025 \tag{5.8}$$

From (5.1), the corresponding equation to be solved is

$$p^2 - q^2 - 6pq + q^2 = 2025$$

$$\Rightarrow p^2 - 6pq = 2025$$

Treating the above equation as a quadratic in p and solving for the same, we get

$$p = 3[q + \sqrt{q^2 + 225}] \tag{5.9}$$

The square-root on the R.H.S. of (5.9) is eliminated when

$$q = 8, 20, 36, 112 \text{ and the respective values of } p \text{ are}$$

$$p = 75, 135, 225, 675$$

The sides of the corresponding Pythagorean triangle are presented below:

$$(x, y, H) = (1200, 5561, 5689), (5400, 17825, 18625), (16200, 49329, 50625),$$

$$(151200, 443081, 868169)$$

It is observed that, for each of the above Pythagorean triangles, the above relation (5.8) is satisfied.

Characterization 5.5

Assume

$$3y - 2H - 4\frac{A}{P} = 2025 \tag{5.10}$$

From (5.1) & (5.2), the corresponding equation to be solved is

$$\begin{aligned}
& 3(p^2 - q^2) - 2(p^2 + q^2) - 4\left[\frac{pq(p^2 - q^2)}{2p(p+q)}\right] = 2025 \\
& \Rightarrow p^2 - 5q^2 - 2q(p - q) = 2025 \tag{5.11} \\
& \Rightarrow (p - q)^2 - (2q)^2 = 2025 \\
& \Rightarrow (p - 3q)(p + q) = 2025
\end{aligned}$$

After performing a few calculations, the values of p, q satisfying (5.11) and values of y, H, A, P are given in Table 5.5 below:

Table -5.5-Pythagorean triangle with $3y - 2H - 4\frac{A}{P} = 2025$

p	q	y	H	$4\frac{A}{P}$	$3y - 2H - 4\frac{A}{P} = 2025$
1519	506	2051325	2563397	1025156	2025
507	168	228825	285273	113904	2025
305	100	83025	103025	41000	2025
171	54	26325	32157	12636	2025
105	30	10125	11925	4500	2025
67	14	4293	4685	1484	2025
63	12	3825	4113	1224	2025

Chapter 6: Special Pairs of Pythagorean Triangles in Relation to Jarasandha Numbers

Let PT_1, PT_2 be two distinct Pythagorean triangles with generators m, q ($m > q > 0$) and p, q ($p > q > 0$) respectively. Let P_1, P_2 be the perimeters and A_1, A_2 be the areas of the Pythagorean triangles PT_1, PT_2 respectively. Then, we have

$$\begin{aligned} A_1 &= mq(m^2 - q^2), A_2 = pq(p^2 - q^2), \\ P_1 &= 2m(m + q), P_2 = 2p(p + q). \end{aligned} \tag{6.1}$$

Pattern 1

Assume

$$P_1 - P_2 = \text{two times the two digits Jarasandha number } 81$$

From (6.1), the corresponding equation to be solved is

$$\begin{aligned} 2m(m + q) - 2p(p + q) &= 2 * 81 \\ \Rightarrow (m - p)(m + p + q) &= 81 \end{aligned} \tag{6.2}$$

After performing a few calculations, the values of m, p, q satisfying (6.2) and the respective perimeters are presented in Table- 6.1 below :

Table-6.1- Values of q, p, m, P_1, P_2

q	p	m	P_1	P_2	$\frac{P_1 - P_2}{2}$
2	39	40	3360	3198	81
2	11	14	448	286	81
4	38	39	3354	3192	81
4	10	13	442	280	81

6	37	38	3344	3182	81
6	9	12	432	270	81
8	36	37	3330	3168	81
10	35	36	3312	3150	81
12	34	35	3290	3128	81
14	33	34	3264	3102	81
16	32	33	3234	3072	81
18	31	32	3200	3038	81
20	30	31	3162	3000	81
22	29	30	3120	2958	81
24	28	29	3074	2912	81
26	27	28	3024	2862	81

Pattern 2

Assume

$$P_1 - P_2 = \text{Six times the two digits Jarasandha number } 81$$

From (6.1) , the corresponding equation to be solved is

$$\begin{aligned}
 2m(m+q) - 2p(p+q) &= 6*81 \\
 \Rightarrow (2m+q)^2 - (2p+q)^2 &= 972 \\
 \Rightarrow (m-p)(m+p+q) &= 243
 \end{aligned}
 \tag{6.3}$$

After performing a few calculations, the values of m, p, q satisfying (6.3) and the respective perimeters are presented in Table- 6.2 below :

Table- 6.2- Values of q,p,m,P₁,P₂

q	p	m	P ₁	P ₂	$\frac{P_1 - P_2}{6}$
2	120	121	29766	29280	81
2	38	41	3526	3040	81
4	119	120	29760	29274	81
4	37	40	3520	3034	81
6	118	119	29750	29264	81
6	36	39	3510	3024	81
8	117	118	29736	29250	81
8	35	38	3496	3010	81
10	116	117	29718	29232	81

10	34	37	3478	2992	81
12	115	116	29696	29210	81
12	33	36	3456	2970	81
14	114	115	29670	29184	81
14	32	35	3430	2944	81
16	113	114	29640	29154	81
16	31	34	3400	2914	81
18	112	113	29606	29120	81
18	30	33	3366	2880	81
20	111	112	29568	29082	81
20	29	32	3328	2842	81
22	110	111	29526	29040	81
22	28	31	3286	2800	81
24	109	110	29480	28994	81
24	27	30	3240	2754	81
26	108	109	29430	28944	81
28	107	108	29376	28890	81
30	106	107	29318	28832	81
32	105	106	29256	28770	81
34	104	105	29190	28704	81
36	103	104	29120	28634	81
38	102	103	29046	28560	81
40	101	102	28968	28482	81
42	100	101	28886	28400	81
44	99	100	28800	28314	81
46	98	99	28710	28224	81
48	97	98	28616	28130	81
50	96	97	28518	28032	81
52	95	96	28416	27930	81
54	94	95	28310	27824	81
56	93	94	28200	27714	81
58	92	93	28086	27600	81
60	91	92	27968	27482	81
62	90	91	27846	27360	81
64	89	90	27720	27234	81
66	88	89	27590	27104	81
68	87	88	27456	26970	81
70	86	87	27318	26832	81
72	85	86	27176	26690	81

74	84	85	27030	26544	81
76	83	84	26880	26394	81
78	82	83	26726	26240	81
80	81	82	26568	26082	81

Pattern 3

Assume

$$P_1 - P_2 = \text{two times the four digits Jarasandha number } 2025$$

From (6.1) , the corresponding equation to be solved is

$$\begin{aligned}
 2m(m+q) - 2p(p+q) &= 2 * 2025 \\
 \Rightarrow (m-p)(m+p+q) &= 2025
 \end{aligned}
 \tag{6.4}$$

After performing a few calculations, the values of m, p ,q satisfying (6.4) and the respective perimeters are presented in Table- 6.3 below :

Table- 6.3- Values of q,p,m,P₁,P₂

S.No	q	p	m	P ₁	P ₂	$\frac{P_1 - P_2}{2}$
1	2 s	1012-s	1013-s	2 (1013 ² -s ²)	2 (1012 ² -s ²)	2025
2	2 s	336-s	339-s	2 (339 ² -s ²)	2 (336 ² -s ²)	2025
3	2 s	200-s	205-s	2 (205 ² -s ²)	2 (200 ² -s ²)	2025
4	2 s	108-s	117-s	2 (117 ² -s ²)	2 (108 ² -s ²)	2025
5	2 s	60-s	75-s	2 (75 ² -s ²)	2 (60 ² -s ²)	2025
6	2 s	28-s	53-s	2 (53 ² -s ²)	2 (28 ² -s ²)	2025

As $m > q > 0$ and $p > q > 0$, It is to be noted that, the boundary values for s in Table 6.3 are presented in Table -6.4 below

Table- 6.4: Boundary for s

S.No	Boundary for s
1	$1 \leq s \leq 337$
2	$1 \leq s \leq 111$
3	$1 \leq s \leq 66$
4	$1 \leq s \leq 35$

5	$1 \leq s \leq 19$
6	$1 \leq s \leq 9$

Pattern 4

Assume

$$P_1 - P_2 = \text{two times the four digits Jarasandha number } 3025$$

From (6.1), the corresponding equation to be solved is

$$\begin{aligned}
 2m(m+q) - 2p(p+q) &= 2 * 3025 \\
 \Rightarrow (m-p)(m+p+q) &= 3025
 \end{aligned}
 \tag{6.5}$$

After performing a few calculations, the values of m, p, q satisfying (6.5) and the respective perimeters are presented in Table- 6.5 below :

Table-6.5- Values of q,p,m,P₁,P₂

S.No	q	p	m	P ₁	P ₂	$\frac{P_1 - P_2}{2}$
1	2 s	1512-s	1513-s	$2 (1513^2 - s^2)$	$2 (1512^2 - s^2)$	3025
2	2 s	300-s	305-s	$2 (305^2 - s^2)$	$2 (300^2 - s^2)$	3025
3	2 s	132-s	143-s	$2 (143^2 - s^2)$	$2 (132^2 - s^2)$	3025
4	2 s	48-s	73-s	$2 (73^2 - s^2)$	$2 (48^2 - s^2)$	3025

As $m > q > 0$ and $p > q > 0$, It is to be noted that, the boundary values for s in Table 6.5 are presented in Table - 6.6 below

Table- 6.6: Boundary for s

S.No	Boundary for s
1	$1 \leq s \leq 503$
2	$1 \leq s \leq 99$
3	$1 \leq s \leq 43$
4	$1 \leq s \leq 15$

Pattern 5

Assume

$$P_1 - P_2 = \text{Two times the difference between two Jarasandha numbers}$$

$$= 2(2025-81) = 3888$$

From (6.1), the corresponding equation to be solved is

$$\begin{aligned} 2m(m+q) - 2p(p+q) &= 3888 \\ \Rightarrow (m-p)(m+p+q) &= 1944 \end{aligned} \tag{6.6}$$

After performing a few calculations, the values of m, p, q satisfying (6.6) and the respective perimeters are presented in Table- 6.7 below :

Table-6.7- Values of q, p, m, P_1, P_2

S.No	q	p	m	P_1	P_2	$P_1 - P_2$
1	2s-1	972-s	973-s	2(973-s)* (972+s)	2(972-s)* (971+s)	2(2025-81)
2	2s	485-s	487-s	2(487-s)* (487+s)	2(485-s)* (485+s)	2(2025-81)
3	2s-1	323-s	326-s	2(326-s)* (325+s)	2(323-s)* (322+s)	2(2025-81)
4	2s	241-s	245-s	2(245-s)* (245+s)	2(241-s)* (241+s)	2(2025-81)
5	2s	159-s	165-s	2(165-s)* (165+s)	2(159-s)* (159+s)	2(2025-81)
6	2s-1	118-s	126-s	2(126-s)* (125+s)	2(118-s)* (117+s)	2(2025-81)
7	2s-1	104-s	113-s	2(113-s)* (112+s)	2(104-s)* (103+s)	2(2025-81)
8	2s	75-s	87-s	2(87-s)* (87+s)	2(75-s)* (75+s)	2(2025-81)
9	2s	45-s	63-s	2(63-s)* (63+s)	2(45-s)* (45+s)	2(2025-81)
10	2s-1	29-s	53-s	2(53-s)* (52+s)	2(29-s)* (28+s)	2(2025-81)
11	2s-1	23-s	50-s	2(50-s)* (49+s)	2(23-s)* (22+s)	2(2025-81)
12	2s	9-s	45-s	2(45-s)*	2(9-s)*	2(2025-81)

				(45+s)	(9+s)	
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As $m > q > 0$ and $p > q > 0$, It is to be noted that, the boundary values for s in Table 6.7 are presented in Table -6.8 below

Table- 6.8: Boundary for s

S.No	Boundary for s
1	$1 \leq s \leq 324$
2	$1 \leq s \leq 161$
3	$1 \leq s \leq 107$
4	$1 \leq s \leq 80$
5	$1 \leq s \leq 52$
6	$1 \leq s \leq 39$
7	$1 \leq s \leq 34$
8	$1 \leq s \leq 24$
9	$1 \leq s \leq 14$
10	$1 \leq s \leq 9$
11	$1 \leq s \leq 7$
12	$1 \leq s \leq 2$

Chapter 7: Pairs Of Rectangles in Connection with Jarasandha Numbers

Let R_1, R_2 be two rectangles with dimensions x, y & z, w respectively.

Let A_1, A_2 be the areas of R_1, R_2 correspondingly.

Choice 1

Assume

$$A_1 - A_2 = \text{two digits Jarasandha number} = 81$$

The above relation leads to the Diophantine equation

$$x y - z w = 81 \tag{7.1}$$

The insertion of the linear substitutions

$$x = u + v, y = u - v, z = t + v, w = t - v, u, t \neq v \tag{7.2}$$

in (7.1) gives the non-homogeneous binary quadratic equation

$$u^2 - t^2 = 81 \tag{7.3}$$

After performing a few calculations, the values of u, t satisfying (7.3) and the sides of the corresponding rectangle are presented in Table- 7. 1 below:

Table-7.1- Rectangles with $A_1 - A_2 = 81$

S.No	u	t	x	y	z	w	$A_1 - A_2$
1	41	40	$41 + v$	$41 - v$	$40 + v$	$40 - v$	81
2	15	12	$15 + v$	$15 - v$	$12 + v$	$12 - v$	81

It is to be noted that, the boundary values for v in Table 7.1 are presented in Table -7.2 below:

Table- 7.2: Boundary for v

S.No	Boundary for v
1	$-39 \leq v \leq 39$
2	$-11 \leq v \leq 11$

Choice 2

Assume

$$A_1 - A_2 = \text{Four digits Jarasandha number} = 2025$$

For this choice ,the corresponding equation to be solved is

$$u^2 - t^2 = 2025 \tag{7.4}$$

After performing a few calculations, the values of u ,t satisfying (7.4) and the sides of the corresponding rectangle are presented in Table-7.3 below:

Table-7.3- Rectangles with $A_1 - A_2 = 2025$

S.No	u	t	x	y	z	w	$A_1 - A_2$
1	1013	1012	$1013+v$	$1013-v$	$1012+v$	$1012-v$	2025
2	339	336	$339+v$	$339-v$	$336+v$	$336-v$	2025
3	205	200	$205+v$	$205-v$	$200+v$	$200-v$	2025
4	117	108	$117+v$	$117-v$	$108+v$	$108-v$	2025
5	75	60	$75+v$	$75-v$	$60+v$	$60-v$	2025
6	53	28	$53+v$	$53-v$	$28+v$	$28-v$	2025
7	51	24	$51+v$	$51-v$	$24+v$	$24-v$	2025

It is to be noted that, the boundary values for v in Table 7.3 are presented in Table – 7.4 below

Table- 7.4: Boundary for v

S.No	Boundary for v
1	$-1011 \leq v \leq 1011$
2	$-335 \leq v \leq 335$
3	$-199 \leq v \leq 199$
4	$-107 \leq v \leq 107$
5	$-59 \leq v \leq 59$
6	$-27 \leq v \leq 27$
7	$-23 \leq v \leq 23$

Choice 3

Assume

$$A_1 - A_2 = \text{Four digits Jarasandha number} = 3025$$

For this choice, the corresponding equation to be solved is

$$u^2 - t^2 = 3025 \quad (7.5)$$

After performing a few calculations, the values of u, t satisfying (7.5) and the sides of the corresponding rectangle are presented in Table- 7.5 below:

Table - 7.5- Rectangles with $A_1 - A_2 = 3025$

S.No	u	t	x	y	z	w	$A_1 - A_2$
1	1513	1512	$1513+v$	$1513-v$	$1512+v$	$1512-v$	3025
2	305	300	$305+v$	$305-v$	$300+v$	$300-v$	3025
3	143	132	$143+v$	$143-v$	$132+v$	$132-v$	3025
4	73	48	$73+v$	$73-v$	$48+v$	$48-v$	3025

It is to be noted that, the boundary values for v in Table 7.5 are presented in Table – 7.6 below

Table- 7.6: Boundary for v

S.No	Boundary for v
1	$-1511 \leq v \leq 1511$
2	$-299 \leq v \leq 299$
3	$-131 \leq v \leq 131$
4	$-47 \leq v \leq 47$

Choice 4

Assume

$$A_1 - A_2 = \text{Four digits Jarasandha number} = 9801$$

For this choice, the corresponding equation to be solved is

$$u^2 - t^2 = 9801 \tag{7.6}$$

After performing a few calculations, the values of u ,t satisfying (7.6) and the sides of the corresponding rectangle are presented in Table- 7. 7 below:

Table-7.7- Rectangles with $A_1 - A_2 = 9801$

S.No	u	t	x	y	z	w	$A_1 - A_2$
1	4901	4900	4901+v	4901-v	4900+v	4900-v	9801
2	1635	1632	1635+v	1635-v	1632+v	1632-v	9801
3	549	540	549+v	549-v	540+v	540-v	9801
4	451	440	451+v	451-v	440+v	440-v	9801
5	195	168	195+v	195-v	168+v	168-v	9801
6	165	132	165+v	165-v	132+v	132-v	9801
7	101	20	101+v	101-v	20+v	20-v	9801

It is to be noted that, the boundary values for v in Table 7.7 are presented in Table – 7.8 below

Table- 7.8: Boundary for v

S.No	Boundary for v
1	$-4899 \leq v \leq 4899$
2	$-1631 \leq v \leq 1631$
3	$-539 \leq v \leq 539$
4	$-439 \leq v \leq 439$
5	$-167 \leq v \leq 167$
6	$-131 \leq v \leq 131$
7	$-19 \leq v \leq 19$

Chapter 8: Linear Combination of Two Polygonal Numbers of Different Size and of Different Ranks in Connection with Special Jarasandha Numbers

Connection 8.1

The assumption

$$4 * t_{3,n} - t_{6,m} - 3m = 4 * 81 \tag{8.1}$$

is equivalent to

$$\begin{aligned} 4\left[\frac{n(n+1)}{2}\right] - (2m^2 - m) - 3m &= 324 \\ \Rightarrow n(n+1) - m(m+1) &= 162 \\ \Rightarrow (n-m) + (n^2 - m^2) &= 162 \\ \Rightarrow (n-m)(n+m+1) &= 162 \end{aligned} \tag{8.2}$$

The values of n, m satisfying (8.2) and the corresponding values of $t_{3,n}, t_{6,m}$ satisfying (8.1) are given in Table 8.1 below:

Table 8.1: values of $n, m, t_{3,n}, t_{6,m}$

n	m	$t_{3,n}$	$t_{6,m}$	$\frac{4 * t_{3,n} - t_{6,m} - 3m}{4}$
81	80	3321	12720	81
41	39	861	3003	81
28	25	406	1225	81
16	10	136	190	81
13	4	91	28	81

Connection 8.2

The assumption

$$4 * t_{3,n} - t_{6,m} - 3m = 4 * 2025 \tag{8.3}$$

is equivalent to

$$\begin{aligned} 4\left[\frac{n(n+1)}{2}\right] - (2m^2 - m) - 3m &= 8100 \\ \Rightarrow n(n+1) - m(m+1) &= 4050 \\ \Rightarrow (n-m) + (n^2 - m^2) &= 4050 \\ \Rightarrow (n-m)(n+m+1) &= 4050 \end{aligned} \tag{8.4}$$

The values of n, m satisfying (8.4) and the corresponding values of $t_{3,n}, t_{6,m}$ satisfying (8.3) are given in Table 8.2 below:

Table 8.2: values of $n, m, t_{3,n}, t_{6,m}$

n	m	$t_{3,n}$	$t_{6,m}$	$\frac{4 * t_{3,n} - t_{6,m} - 3m}{4}$
2025	2024	2051325	8191128	2025
1013	1011	513591	2043231	2025
676	673	228826	905185	2025
407	402	83028	322806	2025
340	334	57970	222778	2025
229	220	26335	96580	2025
207	197	21528	77815	2025
142	127	10153	32131	2025
121	103	7381	21115	2025
93	68	4371	9180	2025
88	61	3916	7381	2025
82	52	3403	5356	2025
67	22	2278	946	2025
65	15	2145	435	2025
64	10	2080	190	2025

Connection 8.3

The assumption

$$2(t_{8,n} - n) - 3(t_{6,m} - m) = 12 * 81 \quad (8.5)$$

is equivalent to

$$\begin{aligned} 2[3n^2 - 3n] - 3(2m^2 - 2m) &= 972 \\ \Rightarrow n(n-1) - m(m-1) &= 162 \\ \Rightarrow (n^2 - m^2) - (n - m) &= 162 \\ \Rightarrow (n - m)(n + m - 1) &= 162 \end{aligned} \quad (8.6)$$

The values of n, m satisfying (8.6) and the corresponding values of $t_{8,n}, t_{6,m}$ satisfying (8.5) are given in Table 8.3 below:

Table 8.3: values of $n, m, t_{8,n}, t_{6,m}$

n	m	$t_{8,n}$	$t_{6,m}$	$\frac{2(t_{8,n} - n) - 3(t_{6,m} - m)}{12}$
82	81	20008	13041	81
42	40	5208	3160	81
29	26	2465	1326	81
17	11	833	231	81
14	5	560	45	81

Chapter 9: Linear Combination of Two Centered Polygonal Numbers of Different Size and of Different Ranks in Connection with Special Jarasandha Numbers

Relation 9.1

The assumption

$$5 Ct_{3,n} - 3 Ct_{5,m} - 2 = 2025 \tag{9.1}$$

is equivalent to

$$\begin{aligned} 5\left[\frac{3n(n+1)}{2} + 1\right] - 3\left[\frac{5m(m+1)}{2} + 1\right] - 2 &= 2025 \\ \Rightarrow n(n+1) - m(m+1) &= 270 \\ \Rightarrow (n-m)(n+m+1) &= 270 \end{aligned} \tag{9.2}$$

The values of n, m satisfying (9.2) and the corresponding values of $Ct_{3,n}, Ct_{5,m}$ satisfying (9.1) are given in Table 9.1 below:

Table 9.1: values of $n, m, Ct_{3,n}, Ct_{5,m}$

n	m	$Ct_{3,n}$	$Ct_{5,m}$	$5 Ct_{3,n} - 3 Ct_{5,m} - 2$
135	134	27541	45226	2025
68	66	7039	11056	2025
46	43	3244	4731	2025
29	24	1306	1501	2025
25	19	976	951	2025
19	10	571	276	2025
18	8	514	181	2025
16	1	409	6	2025

Relation 9.2

The assumption

$$2Ct_{3,n} - Ct_{6,m} - 1 = 6 * 81 \quad (9.3)$$

is equivalent to

$$\begin{aligned} 2\left[\frac{3n(n+1)}{2} + 1\right] - \left[\frac{6m(m+1)}{2} + 1\right] - 1 &= 6 * 81 \\ \Rightarrow 3n(n+1) - 3m(m+1) &= 486 \\ \Rightarrow (n-m)(n+m+1) &= 162 \end{aligned} \quad (9.4)$$

The values of n, m satisfying (9.4) and the corresponding values of $Ct_{3,n}, Ct_{6,m}$ satisfying (9.3) are given in Table 9.2 below:

Table 9.2: values of $n, m, Ct_{3,n}, Ct_{6,m}$

n	m	$Ct_{3,n}$	$Ct_{6,m}$	$\frac{2Ct_{3,n} - Ct_{6,m} - 1}{6}$
81	80	9964	19441	81
41	39	2584	4681	81
28	25	1219	1951	81
16	10	409	331	81
13	4	274	61	81

Relation 9.3

The assumption

$$Ct_{8,n} - 2Ct_{4,m} + 1 = 8 * 81 \quad (9.5)$$

is equivalent to

$$\begin{aligned} \left[\frac{8n(n+1)}{2} + 1\right] - 2\left[\frac{4m(m+1)}{2} + 1\right] + 1 &= 8 * 81 \\ \Rightarrow 4n(n+1) - 4m(m+1) &= 648 \\ \Rightarrow (n-m)(n+m+1) &= 162 \end{aligned} \quad (9.6)$$

The values of n, m satisfying (9.6) and the corresponding values of $Ct_{8,n}, Ct_{4,m}$ satisfying (9.5) are given in Table 9.3 below:

Table 9.3: values of $n, m, Ct_{8,n}, Ct_{4,m}$

n	m	$Ct_{8,n}$	$Ct_{4,m}$	$\frac{Ct_{8,n} - 2 Ct_{4,m} + 1}{8}$
81	80	26569	12961	81
41	39	6889	3121	81
28	25	3249	1301	81
16	10	1089	221	81
13	4	729	41	81

Conclusion:

In this book, we have presented some fascinating relations connecting figurate numbers, Pythagorean triangles and rectangles with Jarasandha numbers. The readers interested in studying patterns in numbers may be motivated to obtain other forms of number patterns.

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