CHAPTER 1

INTRODUCTION

Lie group of transformations and their employment in search of solutions of differential equations dates back to the seminal works of Sophus Lie, a Norwegian mathematician in the last quarter of 19^{th} century. Detailed information about his life and work has been summarized by Fritzsche (1999). The theory is a unification of many ad hoc mathematical solution methods which outlines a deeper understanding of the properties of differential equations and their so called symmetries. In fact, a generalized and unified excellent method with a strong theoretical background has been presented by Sophus Lie (1893). The importance of the method was not fully understood and the work of Lie remained almost in the shadows until 1960s. From then, the importance of the generalized approach, especially being the only general method in attacking nonlinear differential equations in search of analytical solutions, was more realized and used as an efficient tool to handle differential equations. The mathematical models in vast areas such as heat transfer, fluid mechanics, vibrations, elasticity, plasticity, dynamics, quantum mechanics, optics, administrative sciences were successfully solved by employment of the symmetry methods. See the handbooks edited by Ibragimov (1993, 1995, 1996) for a partial but extensive review of the work up to 1996 on the topic as well as original work of Sophus Lie. It is common to collect analytical and numerical techniques for solving differential equations in the form of handbooks (Zwillinger, 1989) and the symmetry techniques find a place although very brief in such general handbooks. Compared to the power and utility of the techniques, symmetry methods still did not receive enough appreciation from the scientific community. The high and advanced theoretical background prevents many people to enter the subject. A simplified and more readable presentation of the topic might be useful to increase the number of researchers employing the methods. At least, some special transformations can be augmented into the differential equation courses at the undergraduate level. This book is intended to convey the techniques to a broader audience and may be considered as an initiate to attract more researchers to the field who are desperate in solving their physical problems.

Apart from this introductory and simplified book on the topic, there are excellent books for deeper understanding of the symmetry methods (Bluman and Kumei, 1989; Olver, 1986; Ibragimov, 1993, 1995, 1996, 1999; Stephani, 1989; Emanuel, 2001; Cohen, 1911). An extensive review of the symbolic computational programs was given by Hereman (Ibragimov, 1996, Part III, Computational Methods). Computational algorithms employing Mathematica was presented by Baumann (2000). For the exterior calculus approach of symmetry methods, an excellent book is due to Şuhubi (2010). The major drawback of the book is that it is written in Turkish which prevents it to reach a wider audience.

Chapter 2 is devoted to special Lie group transformations. It is a brief summary of the general theory presented in Chapters 3-9 for special transformations. Chapter 2 is self-contained and includes the essentials of the theory albeit for special transformations. Even with the mere knowledge of Chapter 2, much can be done for solving differential equations. Reading the chapter is strongly recommended to grasp the ideas presented in the subsequent chapters for outlining the general theory. Chapter 3 is an introduction to the general concepts of the theory such as Lie groups, infinitesimal generators and Lie algebras. In Chapter 4, the symmetries of ordinary differential equations are calculated. Those symmetries are used to construct solutions of ODEs in Chapter 5. Chapter 6 is devoted to the calculation of symmetry generators of the partial differential equations which are used to derive solutions in Chapter 7. For equations having arbitrary parameters and/or functions other than the dependent variables, the group classification problem is treated in Chapter 8. Chapter 9 deals with boundary value problems and discuss the restrictions imposed by the initial and boundary conditions on the symmetries. The last two chapters are about the so called approximate symmetry theories. They may prove to be useful if the equation does not admit exact analytical solutions. The relatively older three approximate symmetry methods were discussed in Chapter 10 and compared with each other. Chapter 11 is about the recently proposed new three approximate symmetry theories and their comparisons with each other and with the exact symmetries.

It is hard to cover the extensive literature and different methods in this introductory book. Only the essentials of the theory are presented for applied oriented researchers with theory kept to a minimum. References for further reading and more advanced methods are mentioned at the end of the book under the title “further topics”.