

Chapter 12: Tensor Products and Functional Calculus of Polynomially Paranormal Operators

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Abstract: This chapter extends the concept of polynomially paranormal operators on Hilbert spaces and their tensor products to the broader class of polynomially $*$ -paranormal and uncertainty-based operators. We introduce new definitions and establish sufficient conditions under which the tensor product of such operators preserves paranormality and spectral stability. Moreover, the single-valued extension property (SVEP) and spectral radius inequalities are investigated for these new operator classes. The results unify operator theoretic structures with uncertainty theory and provide a framework for future extensions to Banach spaces and C^* -algebras.

Keywords: Paranormal operator, Polynomially $*$ -paranormal operator, Tensor product, Uncertainty operator, SVEP, Spectral radius.

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1. Introduction and Literature Review

Operator theory is a fundamental area of functional analysis, concerned with the study of bounded linear operators on Hilbert and Banach spaces. Within this framework, *paranormal operators* occupy a prominent position, generalizing classical concepts such as normal and hyponormal operators while exhibiting rich spectral and topological properties.

The notion of a paranormal operator was introduced by V. Istrătescu in the 1960s [7], and the terminology was later popularized by Furuta. The spectral and topological aspects of paranormal operators were studied in detail by Glenn [6]. Foundational work on hyponormal and quasi-hyponormal operators by Stampfli [10,11] provided motivation for investigating the broader class of paranormal and polynomially paranormal operators.

An operator T is said to be *polynomially paranormal* if there exists a non-constant polynomial $p(z)$ such that $p(T)$ is paranormal. Connections between n -paranormal operators and classical operator classes such as hyponormal or normal operators were explored by Douglas [4] and Cho & Ota [3]. Dowari and Goswami [5] extended these investigations to tensor products of Hilbert spaces, providing sufficient conditions under which the tensor product of paranormal operators remains paranormal.

Further refinements led to the concept of *polynomially $*$ -paranormal operators*, which incorporate adjoint-based conditions to capture symmetries in the operator structure, as studied by Lingling and Uchiyama [8]. This development allows for a more nuanced understanding of operator behavior, particularly in the context of tensor products and spectral theory.

The purpose of this chapter is to unify and extend these notions. We establish results for polynomially $*$ -paranormal operators on tensor products of Hilbert spaces, derive spectral conditions,