

Chapter 7: Operators on Weak Statistical Convergence of Sequences

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Abstract: In this paper, we study the behaviour of linear and nonlinear operators on weakly statistically convergent sequences in Banach spaces. We provide necessary and sufficient conditions for an operator to preserve weak statistical convergence, investigate compositions of such operators, and analyze the effect of compactness and boundedness on the rate of convergence. Counterexamples for unbounded and nonlinear operators are presented, illustrating that weak statistical convergence is not preserved in general. Several results are also established via the adjoint operator, providing a dual space characterization of operator preservation. Applications to function spaces and shift operators are indicated, highlighting the relevance of these results in functional analysis and approximation theory.

Keywords: Weak statistical convergence, bounded linear operator, compact operator, adjoint operator, Banach space, operator composition.

2020 Mathematics Subject Classification. Primary: 40A05, 46B20; Secondary: 47A05, 40G05

Introduction

Weak statistical convergence extends the classical notion of weak convergence in Banach spaces by incorporating the framework of statistical convergence, originally introduced for real sequences by Salát [2]. A sequence (x_n) in a Banach space X is said to be *weakly statistically convergent* to $x \in X$ if for every continuous linear functional $f \in X^*$, the scalar sequence $(f(x_n))$ is statistically convergent to $f(x)$. This concept, first formulated by Bhardwaj and Bala [1], has since become a significant topic of study with applications in operator theory, approximation theory, and the geometry of Banach spaces.

The study of convergence under linear operators is fundamental in functional analysis. Although bounded linear operators are known to preserve weak convergence, it is natural to ask whether they also preserve weak statistical convergence. This question gains importance in light of the growing body of work on statistical convergence of operators (see, for example, [3-6]). Understanding the behaviour of weak statistical convergence under bounded, compact, and adjoint operators provides insight into summability methods, functional approximation, and operator-theoretic structures.