Deep Science Publishing, 2025
https://doi.org/10.70593/978-93-7185-898-4_2

Chapter 2: Fixed Point Theorems in Partial Modular Metric spaces with C-Class Function and Application to Non-linear Volterra Integral Equations

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Abstract: This chapter investigates the existence and uniqueness of fixed-point for mappings endowed with C-class functions within the framework of partial modular metric spaces. In support of the main findings, a detailed example is presented to illustrate the applicability of the main theorem. Motivated by practical implications, we invoke our theoretical outcomes to demonstrate the existence result for solutions of non-linear volterra integral equations.

Keywords: Fixed Point, Partial Modular Metric Space, C-Class Function, Non-linear Volterra Integral Equation.

AMS subject classifications: 47H10; 54H25.

Introduction

The evolution of fixed-point theory in metric spaces originates with the Banach contraction principle (BCP). The BCP is recognized as a fundamental, simple, and classical tool in non-linear analysis.

In [1], Matthews expanded the Banach contraction mapping theorem to the framework of partial metric spaces (PMSs), motivated by application in program verification. Subsequently, various fixed-point results within PMSs have been investigated (see, [2-5]). In [6], Chistyakov formulated the concept of modular metric spaces (MMS)s derived from F-modular and established the foundation theory for this class of spaces. Later, in [7], he established the notion of modular on an arbitrary set and extended the framework to modular metric spaces (also see, [8, 9]). Furthermore, in [10] Hosseinzadeh and Parvaneh introduced a novel generalized metric structure known as partial modular metric spaces (PMMSs), offering new perpectives by integrating features of MMS and PMS. To address the inconsistencies associated with non-zero self distance and triangle inequality, Das et al. [11] refined the concept of PMMS. To date, extensive research has been conducted within this framework, see [12, 13].

In [14], A.H. Ansari established the idea of $C-$class functions, a significant development in fixed-point theory that encompasses a broad class of contractive conditions. Subsequently, in [15], Vashistha et al. established a fixed-point theorem within PMSs by employing C-class functions in conjuction with altering distance functions. Later, Das et al.[11], also employed the concept of C-class functions to establish the existence and uniqueness of fixed-points within PMMSs.

In this chapter, the first section is about the development of PMMSs and C-class function. The second section includes all the basic definitions and properties of the space. The third section presents