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Chapter 1: Bounded Variation of Complex Uncertain Sequences

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Abstract: In this article, we introduce the concept of bounded variation for complex uncertain sequences within the framework of uncertainty theory. The notion extends the classical definition of bounded variation for deterministic sequences to uncertain environments, where uncertainty arises from belief degrees rather than randomness. Fundamental properties of such sequences are established, including convergence criteria and relationships with total uncertain variation. This initial study provides a foundation for further developments in uncertain sequence spaces and their applications in analysis and modelling under indeterminacy.

Keywords: Complex uncertain sequence, bounded variation, uncertainty theory, convergence, uncertain expectation.

1. Introduction

The concept of bounded variation occupies a fundamental position in classical analysis, summability theory, and functional spaces. Introduced by Jordan in 1881, the notion provides a quantitative measure of how much a function or sequence oscillates by examining the total amount of variation in its successive terms. A sequence (or function) of bounded variation exhibits controlled fluctuations, ensuring convergence and integrability properties that are essential in Fourier analysis, approximation theory, and measure theory [1, 2, 6, 7, 8, 9, 10]. In particular, the space of sequences of bounded variation forms a well-structured normed linear space, which plays a pivotal role in the study of convergence and stability of numerical and analytical processes.

In the classical deterministic setting, bounded variation has been extensively studied and generalized to various sequence spaces such as those defined by Cesàro means, almost convergence, and lacunary sequences. These generalizations have deep connections with Banach space theory, orthogonal expansions, and Tauberian theorems. However, many real-world systems—especially in engineering, economics, and decision science—are influenced by factors that cannot be precisely measured or modelled probabilistically. In such contexts, uncertainty arises not from randomness, but from incomplete information, human belief, or subjective assessment.

To handle this type of indeterminacy, Liu [3, 4] proposed U*ncertainty theory*, an axiomatic mathematical framework parallel to probability theory, but grounded on *belief degrees instead* of frequency-based probabilities. The theory is based on four fundamental axioms—normality, duality, subadditivity, and product independence—and provides a consistent structure for analysing uncertain variables, uncertain processes, and their associated expectations. Over the last two decades, uncertainty theory has been successfully applied to diverse domains including uncertain differential equations, uncertain optimization, and uncertain statistics, thereby offering an alternative to stochastic modelling when data or randomness assumptions are inadequate.