



Beyond General Relativity: Critical Perspectives on Gravitation, Curvature, and Wave Propagation in Modern Physics

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DeepScience

Published, marketed, and distributed by:

Deep Science Publishing
USA | UK | India | Turkey
Reg. No. MH-33-0523625
www.deepscienceresearch.com
editor@deepscienceresearch.com
WhatsApp: +91 7977171947

ISBN: 978-93-49910-70-6

E-ISBN: 978-93-49910-00-3

<https://doi.org/10.70593/978-93-49910-00-3>

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Citation: Lavenda, B. (2025). *Beyond General Relativity: Critical Perspectives on Gravitation, Curvature, and Wave Propagation in Modern Physics*. Deep Science Publishing.
<https://doi.org/10.70593/978-93-49910-00-3>

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Dedication

To Fanny

Preface

This is not a cut and paste book. In view of the voluminous literature on the subject of gravitation, that would be pointless. The book does not pretend to claim that difficult concepts can be made easily accessible to babies, dogs, etc., and does not feign to demystify relativity, or anything else. It neither reads Newton's mind nor displays Einstein's confusion.

These are all gimmicks that detract from the real purpose, other than personal gain, of writing a book on gravitation or, for that matter, any scientific book. Other books claim to explain reality for not what it appears to be, or to shine light on what is hidden in plain sight. While other popular books try to lull the reader to sleep by singing black hole blues, or jazzing up physics, or explaining gravitation in 'quirky' banana analogies. These should be, really, considered as insults to the lay reader's intelligence.

No one really knows what gravity is, neither the engineers at the LIGO and VIRGO facilities, nor the mavens in general relativity that cashed in on the Nobel Prize. This reminds me of my mentor's Nobel Prize on dissipative structures, which has all been but forgotten. What is certain is that numerical relativity, upon which LIGO and VIRGO draw their spectra, is reducing what they believe to be 'general' relativity to a Le Sage-type theory that is easily refutable.

This book continues where its predecessor, *Seeing Gravity*, also self-published for obvious reasons, left off. The reader may find it repetitive in some sections, and concepts are used which are developed more fully in parts of the book. As such, it vaguely similar to a book that Clerk-Maxwell or Lord Rayleigh would write where different concepts are separated by numeration, without intending any strict continuity. However, these small points shouldn't be a cause of concern.

The references are given by titles because there is no need to cite the journals when the articles are easily retrievable on the internet. This is one good thing that technology has achieved; to rid the shackles of getting papers accepted to peer-reviewed journals. But, now another mafia has arisen in its place, Cornell's *arXiv* staff who are nothing other than a bunch of computer technicians that profess expertise in all that is submitted to their repository, while clearly demonstrating their complete lack of knowledge

of scientific matters. They filter out what the establishment tells them to. Science is not determined by a consensus among majority; there is no democracy in science.

Committees don't crown scientific achievements, and peer-consensus doesn't make theories right. There is no stamp of purity in science, and there is no final word. That is something left for future generations to determine, not those in the midst of the turmoil. Scientific theories must be left to age like good wine before their full-flavor has been reached. The same opinion was echoed by Max Planck when he said that scientific revolutions don't occur by upheavals, rather, they occur when one generation dies out, and is replaced by a new generation that is ignorant of old prejudices. So it was with the quantum revolution, and, so too, will it be with a theory of gravitation. There is no 'final' theory, or theory of 'everything.' As we keep progressing we will always keep learning. That's the beauty of science as opposed to religion.

Parenthetically I would add that my first book, *Thermodynamics of Irreversible Processes* was written almost half a century ago on an IBM ball electric typewriter. The original manuscript had to be sent from Napoli by surface mail to the publisher in Basingstoke. At the time, I was out of work for having criticized a Nobel Laureate who happened to be my thesis advisor. But this did not dissuade my editor, H Holt, at Macmillan Press, from publishing—and even advertising my book on the opposite page of the review of Prigogine & Glansdorff's *Thermodynamic Theory of Structure, Stability, and Fluctuations*.

Half a century later, the typewriter is gone, being replaced by a PC with an overleaf template, the post has been replaced by email, and the publisher has been superseded by independent publishing. And the subject of the Nobel prize has fallen into relative obscurity. Now, I'm retired, but still critical of Nobel Laureates, and their cohorts. But, thank heavens, science knows of no age!

Bernard Lavenda

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1 Introduction

Of all the branches of physics, the theory of gravity has made negative progress. This is much like the negative energy that is stored in the gravitational field in contrast to the positive energy stored in the electrodynamic field.

Newton was content to describe gravity by the falling apple; Einstein was not completely content with the statement that gravity is geometry. Gone is the apple, and the force that makes the apple fall to the ground. In its place there is a trampoline which indent when mass is placed on it like in Fig. (1.1). The rub, however, is that what is pulling the mass down is still the *force* of gravity. Whether we treat a falling apple or a sagging mass, its all the same.

Over the centuries, gravity has succeeded in evading an explanation of what it really is and how fast it propagates, so Newton contented himself with a mere description of what it is. Einstein, on the other hand, buried its properties in a metric of spacetime. The trite saying that gravity is geometry void of meaning because the fabric of space-time bends and distorts even without mass. That's like saying there is an electrostatic field even in the absence of charge.

Gravitational energy poses a problem that was appreciated by James Clerk-Maxwell himself. Having placed the crown on his field equations of electromagnetism, he proceeded to extend such a field interpretation to gravity. However, unlike electromag-

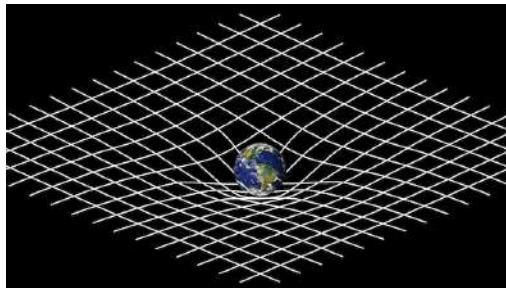


Figure 1.1: Pictorial way of showing how mass indents space and time. Yet, what indents the fabric of space-time if not the downward pull of the gravitational force?

netic energy that can be stored in the field, Maxwell was stopped in his tracks to find that the field stores negative energy. And unlike electromagnetic interactions which can be both positive and negative, gravitational interactions are always positive until it was found that it could become repulsive at high velocities. But, no one was able to harness such energy let alone observe it.

Einstein's field theory equates geometry to physics. The geometry is in the form of Riemann's tensor, and the physics is in the form of an energy-stress tensor. Although the geometry accounts for gravitational interactions, the energy-stresses which cause them must be devoid of gravitational energy. Yet, the energy-stress tensor contains mass, the source of gravity so as to render the separation meaningless.

Gravity is reserved for a pseudo-tensor which can be made to disappear through a judicious choice of the coordinates. Gone was his dream of creating a covariant theory, one that would be valid for what ever coordinates were chosen. So as not to abandon all hope, Einstein considered the possibility of the vanishing of the pseudo-tensor be a statement of his principle of equivalence. Just as acceleration can be annulled by free-falling elevators, the pseudo-tensor can be made to vanish by a mere change in the coordinates. Yet, the former is a dynamic balancing of opposing forces whereas the latter is a mere change in coordinates.

Rather than looking at the cracks in the theory, focus on the results it has borne fruit to. Yet, all the classic tests of general relativity could be obtained by considering gravity as an optically active diffraction medium. This was known to Eddington in the early twenties, who also realized the catastrophic effects that would occur if gravitation propagated at a finite speed. In particular, it would throw off kilter the orbits of the planets in the solar system, since it would take a finite amount of time for the gravitational force to arrive at a planet, so that the planet would not be at the same place as when the force was emitted.

Eddington¹ explains this very well using Fig. (1.2). If the sun attracts Jupiter to its present position J and Jupiter attracts the sun at S , the two forces will be along the same line and balance each other. But, if the sun attracts Jupiter at its previous position, J' , because it takes time for the force of propagation to reach Jupiter, and so, too, Jupiter

¹A S Eddington, *Space, Time and Gravitation*, 1920, p. 94.

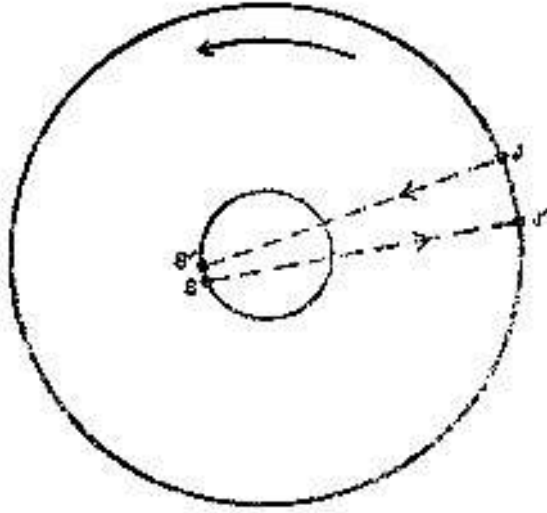


Figure 1.2: The attraction between the sun and Jupiter will not create a couple only when the force that attracts Jupiter to the sun is on the same line as force which attracts the sun to Jupiter.

will attract the sun at its previous position, S' , again because of the finite time to cross the distance between them, a couple will be created that will throw Jupiter's orbit off kilter.

Eddington throws the argument out saying that if these were two electric charges instead of planets, the force will be directed to its present position. We will discuss this latter on in the book, but suffice it to say that it excludes acceleration, and can apply only to uniform motion. The argument used to show that the force is directed to the present position of the moving charge is to show that there is no aberration, but this applies to bodies in uniform motion.

Eddington concludes that in “the theory given in this book, gravitation is propagated with the speed of light, and there is no discordance with observation.” That was in 1920, and certainly no observations were made on the speed of gravity. Even today, there is still no observations on the speed of gravity, notwithstanding what LIGO claims.

If gravitational radiation is emitted with gravitational waves, it is difficult to see how Einstein's field equations account for such dissipative and dispersive properties. Surely, Einstein's condition for the vanishing of the 4-divergence of the Einstein tensor necessarily implies the corresponding condition for the energy-stress tensor. Yet, with the emission of gravitational waves, the conservation of energy and momentum is destroyed.

According to Landau & Lifshitz² dissipative processes like viscosity and thermal conduction can be appended onto the energy-stress tensor, and using the equations of continuity convert the condition of the 4-divergence of the energy-stress tensor into an expression for the 4-divergence of the entropy flux. So conservation has given way to a type of Poynting vector where the 4-divergence is balanced by the processes that lead to an increase entropy. But, even if were to acquiesce to the conversion of an energy conservation condition into a law for the increase in the 4-divergence of the entropy flux, it says nothing about gravitational radiation since the energy-stress tensor excludes gravitational energy *per se*. And if all gravitational interactions lead to the creation of gravitation radiation and the emission of gravitational waves there could never be any stable orbits, for no matter how small the radiation may be its cumulative effects would certainly be discernible of the aeons that the universe has been in existence.

The emission of gravitational radiation any time two bodies interact gravitationally rules out the possibility of stable orbits, orbits which we know exist! And there is, perhaps even more important, the supposed finite propagation of the gravitation force/waves. For if gravitational wave propagation would admit to a finite speed of propagation, the delay caused by the travel time would raise havoc with the clock-work behavior of the planetary orbits, and require phenomena related to diffraction, and, even more important, aberration. Yet, no such phenomena has ever been observed, and probably never will be.

There is no satisfactory way to define an energy density, or even the localization of energy in general relativity. Flat Minkowski space is supposed to have zero energy, yet it is said to be stable against small perturbations because the energy of linearized gravitational waves is purported to be positive. There is even a positive energy theorem

²L D Landau & E M Lifshitz, *Fluid Dynamics* §127.

that states the total energy of a pure gravitational field—in the absence of matter—is always positive! That like talking about positive electromagnetic energy in the absence of charges.

If the gravitational field is the analog of the electric field, what is the analog of the magnetic field? And without a magnetic field how can you talk of the propagation of waves carrying energy and momentum? The linearization of the Einstein equations admits a whole host of indiscretions. One can define a gravitomagnetic vector field, as analogous to a magnetic field, but is this any more than hand waiving? Mathematical entities do not necessarily correspond to physical entities.

The attraction of general relativity is undoubtedly due to what the reader can inject into the dubious, and often unintelligible, nature of the results that it proffers. From the very start, we are told that gravity is geometry. Yet, as the Schwarzschild metric bears testimony, geometry exists even without masses. Would electricity exist in the absence of charges? However, it can be argued that the Schwarzschild metric does possess a mass. Notwithstanding this, O'Neill³ claims that the central mass is taboo—something off limits that is “not to be modelled.” Why then did the mass creep in, in the first place? Through analogy in the weak-field limit where, somehow, geometry morphs into physics: Newtonian gravitation must emerge in the weak-field limit of general relativity, for otherwise, there would be no connection between the two. But it is as if mass is coming out of the woodwork.

Even if we were to accept the presence of a central mass, how can gravitational mass be explained? You need two to tango. For that, the subterfuge of an imaginary ‘test’ mass is brought in. We need another mass to create gravitational attraction. but the mass does not attract anything. How can a central mass attract a peripheral mass without the peripheral mass attracting the central mass?

Up until the sixties, physicists were happy to accept the two solutions that Karl Schwarzschild obtained by solving the Einstein’s field equations one treating a point central mass under the Einstein condition of ‘emptiness,’ and the other which treated a diffuse mass density without the emptiness condition. Emptiness meant to Einstein that his tensor had to vanish, which in an empty universe is equivalent to the vanish-

³B O'Neill, *Semi-Riemann Geometry*

ing of the Ricci tensor. The eigenvalues of the Ricci tensor represent an *average* of the sectional curvatures in any specified direction. This is not to say that all sectional curvatures vanish, but, rather, their sum in any given direction should vanish. Yet, why should the individual sectional curvatures exist, and their sum vanish? If there is no mass, what creates the distortions that show up in the finite sectional curvatures?

Furthermore, if the geometry accounts for gravitational interactions, then setting the divergence of the Einstein tensor equal to zero has nothing whatsoever to do with the vanishing of the divergence of the energy-stress tensor. This is because gravitational energy is not accounted for in the latter.

Yet, when it comes down to the nitty-gritty of the matter, Einstein found it necessary to account for gravity in the form of ‘pseudo-tensor’, which unlike a real tensor, can be made to vanish by a mere coordinate transformation. Einstein attributed this to his equivalence principle between all forms of acceleration and gravity. Just like a person in an elevator feels weightlessness due to the disappearance of gravity, so the effects of gravity can be made to vanish by applying the equivalent acceleration in compensation. This is fine in rectilinear motion, but what about a uniformly rotating disc? There is nothing to compensate the uniform acceleration that an observer would feel on the periphery of the rotating disc.⁴

And Einstein predicted that because of the uniform rotation, his watch would go slower than an observer placed at the center of the disc, which would constitute an inertial frame of reference. However, Einstein’s intuition was not powerful enough to explain why two observers located at different points on the disc cannot discriminate their positions with their rulers and clocks. For, in fact, the rulers would shrink and time would slow down as the inhabitants of the disc became smaller and smaller as they approached the rim of the disc. This makes it all but impossible for the inhabitants to figure out where they are on the disc. All positions on the disc are equivalent because a uniformly rotating disc does not belong to our Euclidean world. It is an example of a hyperbolic world that was discovered by Nicolas Lobachevski and Janos Bolyai, almost a century earlier than the nascent relativity at the turn of the twentieth century.

Einstein’s theory of general relativity is concerned uniquely with geodesic motion;

⁴See, for example, *A New Perspective on Relativity: An Odyssey in Non-Euclidean Geometries*, Ch. 7.

that is, motion that is unaffected by acceleration. In general relativity, ‘proper’ acceleration is considered physical acceleration—that is, the acceleration that an accelerometer would measure. Proper acceleration is the acceleration, in addition to free-fall, that an observer would feel. A cardinal assumption of general relativity is that gravitation does not cause proper accelerations. According to the Wikipedia article on proper acceleration:

Gravitation therefore does not cause proper acceleration, since gravity acts upon the inertial observer that any proper acceleration must depart from. A corollary is that all inertial observers always have a proper acceleration of zero.

Granted, all inertial observers have zero acceleration, but that is not when gravitational forces are present. The distinction is made between proper acceleration and ‘coordinate’ acceleration. Again, according to the Wikipedia article, “[i]n the standard inertial coordinates of special relativity, for unidirectional motion, proper acceleration is the rate of change of proper velocity with respect to coordinate time.” However, special relativity does not treat accelerations since all inertial systems have constant velocity. And why make reference to the special theory insofar as

In an inertial frame in which the object is momentarily at rest, the proper acceleration 3-vector, combined with a zero time-component, yields the object’s four-acceleration, which makes proper-acceleration’s magnitude Lorentz-invariant.

Constant velocity means zero acceleration full stop. Why bring a fourth component of acceleration (which happens to be zero), and claim that the magnitude of proper-acceleration is a Lorentz invariant. All this muddles things rather than adding clarity to a picture which is already blemished by the fact that gravity cannot cause proper accelerations. It can hardly be sustained that “the concept is useful: (i) with accelerated coordinate systems, (ii) at relativistic speeds, and (iii) in curved spacetime.” Curved spacetime is what Einstein attempted to fill the gap that was missing in his flat spacetime of special theory: the accountability of gravitational interactions!

The Wikipedia article is full of double talk. Take for instance the claim

the proper acceleration is the acceleration felt by the occupants [of a vehicle], and which is described as g-force (which is not a force but rather an acceleration; see that article for more discussion of proper acceleration) delivered by the vehicle only.[2] The "acceleration of gravity" ("force of gravity") never contributes to proper acceleration in any circumstances. . .

The reference is to Rindler's *Relativity* which makes the opposite claim that

Proper acceleration is precisely the push we feel when sitting in an accelerating rocket. Also, by the equivalence principle, the gravitational field in our terrestrial lab is the negative of our proper acceleration, our instantaneous rest-frame being an imagined Einstein cabin falling with acceleration g .

The concept of a Lorentz invariant acceleration is also broached on the same page in Rindler's book where he says:

A case of particular interest is that of rectilinear motion with constant proper acceleration α . [An involved procedure] yields the following equation for the motion:

$$x^2 - c^2 t^2 = c^4 / \alpha^2.$$

Thus, for obvious reasons, rectilinear motion with constant proper acceleration is called hyperbolic motion.

The left-hand side of the above equation is used to show that all inertial frames are equivalent, not that the acceleration is a universal constant! What about light where the left-hand side vanishes? Does that mean the acceleration is infinite? Acceleration of what?

Why the test particle idea in the outer Schwarzschild solution has been able to dodge all its critics is due to the fact that in Newtonian gravity, the equivalence of acceleration and gravity depends only on the central, attracting mass; the mass which is attracted to it cancels on both sides of Newton's II.

Regge plots, from which the Eddington and Chandrasekhar masses follow as particular values. We again turn to ellipsoidal configurations to determine stability criteria.

In Chapter 6 we show that what are conics to planetary motion, Cassini ovals are to binaries. Just as slices of a cone result in conic sections, slices of a torus given Cassini ellipses. The similarity in images between Roche lobes and a lemniscate is too good to be missed. The question then is what replaces Newton's laws of conics whose centers coincide with one of their foci. We show that the dual law $[1, -2]$ for planetary motions goes over into the dual law $[-4, -7]$ for Cassini ovals. We then go on to discuss central forces, and, in particular to Bertrand's theorem underlying the fact that the theorem pertains solely to circular orbits which excludes those obtained by slicing up cones and tori. Moreover, we are able to associate the various physical phenomena, like the advance of the perihelion and the deflection of light by a massive body, with specific terms in the law of force. The force will therefore not be entirely a central force, but, rather, consist of small correction terms whose physical effects are clearly discernible.

Much ado has been made of the fact that the square of an ellipse is still an ellipse, just displaced so that was the center of the conic section is displaced to its foci. Rather, it is the geometric mean of the radial coordinate that is displaced when we consider either one of the two elements of the geometric mean. Moreover, we find a problem in the apparent lack of 'reversibility' in the Bohlin-Kasner theorem. What is conserved energy in one transform is not the same as in the reverse transform. This shows that the ellipses are not the same, as we would expect. So it is not just a displacement we get on squaring.

In Chapter 8 we continue our criticisms of general relativity's condition of emptiness in terms of sectional curvatures, and a new quantity, the Schwarzian derivative, or Schwarzian for short, when the third derivative of the curve becomes important for non-symmetrical curves. Stability conditions are shifted from Liapounov exponents to the sign of the Schwarzian.

Hodographs require the notion of "jerk," the time derivative of acceleration. Deviations from Newton's inverse-square law involve the concept of aberrancy. The relation between Kepler's II and III is obtained through Newton's revolving orbit theorem, and

bring in sinus spirals into the discussion. Transforming the logarithmic spiral curve to velocity space gives periodic orbits, and provides a new Hubble-type law that explains the non-decay in the rotational curves of galaxies. The new Hubble relation, expressed in terms of the ratio of the conservation of angular momentum in velocity and configuration space, between acceleration and velocity reduces the decay of acceleration from its Newtonian value of $1/r^2$ to $1/\sqrt{r}$.

In Chapter 9 we conclude our discussion with the parallelisms between general relativistic concept of what a gravitational wave is and Le Sage's shadow theory. Le Sage's "ultra-mundane" particles have been replaced by "gravitons," which have in common, among other things, that both have never been observed. The partial shielding of gravity is related to the dynamical effect of the displacement of the mirrors in the LIGO interferometer: No shielding, no displacement. We continue with Majorana's experimental results, and their seemingly non-reproducibility. The efficiency of gravitational wave absorption is discussed in terms of the Poincaré ratio for the stability of rotating and gravitating ellipsoids, and is shown to have nothing to do with efficiency.

2 The *Principia* Revisited

2.1 A mere question of curvature

Newton's derivation of the inverse-square law and the equation of an elliptical orbit depends crucially on his expression for the radius of curvature. The expression he derived in the early 1670's was

$$\rho = r \frac{(1 + z^2)^{3/2}}{1 + z^2 - z'} \quad (2.1)$$

in polar coordinates, (r, θ) , where $z = (1/r)r'$ is Newton's slope, and the prime indicates differentiation with respect to θ . The derivative of the slope can be expressed as

$$z' = \frac{r}{2} \frac{d}{dr} (1 + z^2),$$

or $(1/z)dz/d\theta = (1/r)dz/dr$.

The denominator of (2.1) will be particularly important since it will be proportional to the product of the force times the square of the radial coordinate. The numerator is essentially a correction factor due to the fact that the curve that Newton considered was not unit speed.

The curvature (2.1) simplifies considerably by introducing $u = 1/r$ in which case it becomes

$$\rho = \frac{(1 + u'^2/u^2)^{3/2}}{u + u''}. \quad (2.2)$$

The complementary angle, θ , is related to the slope by

$$z = \frac{1}{r} \frac{dr}{d\theta} = \tan \theta. \quad (2.3)$$

Thus, the factor appearing in the numerator of the radius of curvature, (2.1) is

$$(1 + z^2)^{3/2} = \sec^3 \theta,$$

and if the denominator is to be constant, then

$$\rho \cos^3 \theta = \text{const.} \quad (2.4)$$

Yet, if we plug in (2.3) into the denominator of (2.1) we find $1 + \tan^2 \theta - \sec^2 \theta = 0$. J B Brackenridge,¹ defines $z = \cot \alpha = \cot(\pi/2 - \theta) = \tan \theta$, undoubtedly not realizing that this sends the radius of curvature to infinity, i.e, a straight line. And if we are given the equation of the orbit as (2.3) why should we look further in order to derive the equation of the orbit? It would be more constructive to consider conic using intrinsic equations.

If we are given a curve, $y = y(x)$, which is continuous and possesses at least a second derivative, we set

$$y' = \tan \theta, \quad (2.5)$$

where the prime stands for differentiation with respect to x . The second derivative is

$$y'' = \sec^2 \theta \theta'.$$

Now, introducing the arc length, s , and its relation to $x = s \cos \theta$, we get the second derivative as

$$y'' = \sec^2 \theta \frac{d\theta}{ds} \frac{ds}{dx} = \sec^2 \theta \cdot \frac{1}{\rho} \cdot \frac{1}{\cos \theta},$$

and we obtain (2.4) if the second derivative is constant. This is certainly not true for all conics, but it is true in Newton's case.

By a special choice of axes, any conic can be written in the form

$$y^2 = A + Bx + Cx^2, \quad (2.6)$$

with the x -axis being the axis of symmetry. The derivatives of (2.6) satisfy

$$yy'' + y'^2 = C. \quad (2.7)$$

Taking the derivative again yields

$$yy''' + 3y'y'' = 0,$$

¹in "The critical role of curvature in Newton's developing dynamics," in *The Investigation of Difficult Things*

Given that all components depend on space and time only through the combination, $x - ct$, the flux reduces to

$$\begin{aligned} t^{01} &= \frac{c^3}{32\pi G} \left[\dot{h}_{23}^2 + \frac{1}{4} (\dot{h}_{33}^2 - \dot{h}_{22}^2) \right], \\ &= \frac{c^3}{16} \left(\frac{\omega^2}{2\pi G} \right) [h_{23}^2 + h_{22}^2], \end{aligned}$$

which is (9.1). To make any sense out of this formula, we have to define a density. The energy density is $t_{00} = \rho c^2$, where ρ is the mass density of the body. Thus, the energy density of the gravitational waves is simply the mass density of the body which emits them and their energy is identified as the rest energy of the body. The pertinent quantity is the ratio

$$t^{01}/t^{00} = \frac{c}{32} \left(\frac{\omega^2}{2\pi G\rho} \right) |a|^2 |\cos\theta|. \quad (9.2)$$

Dyson considers the gravitational waves impinging on the surface of the earth for which the inverse square of the Newtonian free-fall time is $2\pi G\rho \sim 1.25 \times 10^{-6} \text{ sec}^{-2}$. This together with the frequency of a gravitational wave with a 1-sec period, $\omega = 2\pi \text{ sec}^{-1}$, set the Poincaré ratio at

$$\frac{\omega^2}{2\pi G\rho} \approx 10^7.$$

Alternatively, considering a neutron star with mass density 10^{17} kg/m^3 , the square of Newton's free-fall time is of the order 10^{-7} so that the Poincaré ratio is 10^{-6} . This would give an energy flux some 13-orders of magnitude greater for the earth than a neutron star! Clearly, Poincaré's ratio has nothing to do with the energy flux emanating from stellar bodies.

Unfortunately, Marjorana's experiments were never to be repeated, and other confirmations of gravitational shielding were hard to come by.⁶ The attribution of mechanical and dynamic properties to gravitational waves should be foremost in the minds of proponents of numerical relativity. For, it is numerical, and not general, relativity that

⁶M Edwards, Ed. *Pusing Gravity*

has insisted on these attributes. If gravitational waves are to be considered at all, we must look to their dynamical properties, for what else would cause the mirrors to separate in the LIGO interferometer with the passage of a gravitational wave? However, giving gravitational waves attributes like electromagnetic waves is definitely problematical. Electromagnetic waves would be completely shielded from entering the tubes of the interferometer, whereas gravitational waves would undergo only a partial shielding, depending on their absorption coefficient, h . However, their absorption through the tubes of the interferometer must be much greater than what Majorana proposes h to be.

To get an idea of the shielding involved, the universal gravitation constant is expressed in the form ⁷

$$\mathbf{G} = \frac{nm}{4\pi} v^2 h^2,$$

where m is the mass of a single, Le Sage ultra-mundane particle, n their number density, v their speed, and h their cross-sectional area for collisions, or, equivalently, their universal absorption coefficient. Le Sage was cognisant of the fact that h had to be small, for, otherwise, measurements made during a lunar eclipse of the sun would result in a diminution in the gravitational attraction between earth and sun that would make it measurable. By making h small enough, means increasing the number density of particles proportionally, n , and also their velocities, v .

Laplace weighed in on Le Sage's theory since it could be refutable. He reasoned that if the heavenly bodies were moving through a sea of ultra-mundane particles then they would be slowed down, i.e., experience a dragging force. The resistive force that they would experience is

$$F_{res} = \frac{4}{3} M_1 h n m v u,$$

where M_1 is the mass of the body whose velocity is u . The ratio of this resistive force to gravitational attraction requires

$$\frac{F_{res}}{F_{att}} = \frac{u}{v} \frac{r^2}{h M_2} \ll 1, \quad (9.3)$$

where M_2 is the central mass in Newton's law of attraction

$$F_{att} = \mathbf{G} \frac{M_1 M_2}{r^2}.$$

⁷J Evans, "Gravity in a century of light," in *Pushing Gravity*

Since the absorption coefficient is in the denominator of (9.3), the speed, v , of the ultra-mundane particles must be very large. It is this conclusion that Laplace arrived at when he said by imposing (9.3) “would imply in the gravific fluid a speed incomparably greater than that of light, and all the more considerable as the sun and earth leave a freer passage to this fluid.” So a small absorption implies a large velocity for these ultra-mundane particles. However, such large velocities would increase their ‘radiation pressure.’

On another occasion, Laplace reasoned that the speed of gravity must be at least a million times greater than the speed of light. This he did by estimating the time of collision of the earth and moon, based on a mistaken impression that the two were on a collision course.

The partial absorption of gravitational waves would require a velocity well in excess to that of light. So the same conclusion that repelled Laplace to Le Sage’s corpuscular theory would also repel him to gravitational waves.

Undoubtedly, gravitation has resisted explanation due to its unusual properties. Its effect on a body is independent of the affecting body. This is what has allowed general relativity to turn a 2-body problem into an effective 1-body one. The gravitational attraction of the central mass in Schwarzschild’s outer solution does not need an orbiting mass. And if we listen to O’Neill, the central mass is off-limits, something that is not to be modeled. So where does that leave us?

How fast bodies fall in a gravitational field is independent of their masses (Galileo). This is contrary to the law of inertia whereby the greater the mass the greater will be its momentum.

Gravity is able to accelerate bodies with equal ease no matter if they are bound or unbound. Electromagnetism uses electromagnetic waves, and their constituent photons, to transmit their actions. Gravitational attraction, on the other hand, would occur by blocking their elementary transmitters rather than emitting them.

The connection between Le Sage’s theory and general relativity hasn’t gone unno-

ticed.⁸ Shielding in the direction of the body results in a decrease in pressure so that any particle, or (electromagnetic) wave, would fall towards the body, or result in an increase in wavelength, respectively. The pressure variations would be analogous to the curvature of space (but not time). And the increase in wavelength would give the otherwise gravitational field the attribute of another medium with a different index of refraction.

Even Einstein had his doubts about the demise of the ether,

It would have been more correct if I had limited myself, in my earlier publications, to emphasizing only the nonexistence of an ether velocity, instead of arguing the total nonexistence of the ether, for I can see that with the word ether we say nothing else than that space has to be viewed as a carrier of physical qualities.

We have thus come full circuit without a satisfactory explanation of what gravity is and how it operates.

So where do we stand with Le Sage's theory and general relativity which attributes both structural and mechanical properties to gravitational waves? In view of Fig. (9.1), science has been successful in explaining the binding of atoms and molecules through electromagnetic forces, even elementary particles through nuclear forces, but has come no closer to explaining the attraction of gravity. Probably, it is for this reason, and no other, that gravity has attracted so much attention!

⁸M R Edwards, "Le Sage's theory of gravity: The revival by Kelvin and some later developments," in *Pushing Gravity*

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Bernard Lavenda

Beyond General Relativity: Critical Perspectives on Gravitation, Curvature, and Wave Propagation in Modern Physics

What happens when you push a theory beyond its limits? You get a theory like general relativity. Although coined by Einstein, he would be hard pressed to recognize it. Einstein constructed a theory that would pertain to geodesic motion, or motion at constant velocity. Einstein field equations are equivalent to geometrical optics whose characteristic surfaces are those of electromagnetic, and not gravitational, radiation. In fact, gravitational energy is not included in the Einstein energy-stress tensor. As such it excludes catastrophic phenomena like the merger of black holes or neutron stars. In fact, black holes are what you get when you extend a non-Euclidean metric, like the Schwarzschild metric, beyond its domain of validity. Although there exists no solution to Einstein's field equations for two interacting mass points, its numerical counterpart vants at being able to describe binary black hole collisions. Such singularities were pernicious to Einstein's conception of the universe, and he built bridges to avoid them. Nonlinear equations like Einstein's cannot be approximated by their linearization over large portions of spacetime thus placing in doubt the propagation of gravitational waves. Any wave phenomenon traveling at a finite velocity must show signs of aberration. No signs of such have ever been observed. The field equations do not possess a mechanism for the emission of gravitational waves, least of all for their attenuation.

